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An Optimal Reordering Inventory Policy for Deteriorating Products in Fuzzy Set Terminology

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Abstract: This research paper provides a replenishment model for unpreserved products with consideration of the time value of money under a fuzzy situation. This mathematical model follows the discounted cash flow approach to provide inventory replenishment difficulty over a fixed planning horizon. In this model, the authors have shown that the total variable cost is minimized without backlogging under a fuzzy environment. Numerical examples are given to reveal the applicability of the model with respect to the different parameters of the system. This model is developed for uncertain business houses where the businessman is totally dependent on the arrival of the order and the customer; hence, we provided a solution in this paper for those types of business houses.

1. Introduction

Several types of fuzzy inventory models are being discussed in the development of inventory models. Inventory management needs a demand forecasts and parameters for inventory costs for example holding, shortages, replenishment, and backorders. An exact solution of these model property is repeatedly difficult in practice, the inventory linked data can be calibrated using the fuzzy techniques, which facilitates trade with the real-world cases in a more suitable way. There are numerous kinds of deteriorating rate in the present study, such as constant deteriorating rate, deteriorating rate is a linearly increasing function of time, deteriorating rate is two-parameter and three-parameter Weibull distributed. Among them, the constant deteriorating rate is the easiest one and the three-parameter Weibull distribution deteriorating rate is more difficult. Several studies which belong to the first category have made extensive study in this factor. Most of the models during the review, the inflation, and the time value of money unseen. This has happened mostly because of the faith that the inflation and the time value of money would not influence the inventory policy to any significant level. We recommend and build up a fuzzy model for time-varying decaying items and establish optimal replenishments policy corresponding to cycle length during the time horizon H, consisting of positive and negative inventories periods for both with and without shortage to get a more simplify results. In addition, we prove that the total variable cost functions are convex under the fuzzy system. Numerical examples are to reveal the applicability of the proposed models buildup of this article is organized as follows. In 1991, Goswami and Chaudhuri wrote an EOQ model for deteriorating item with shortages and a linear trend in demand. Datta and Pal hub 1999 wrote a note on a replenishment policy for an inventory model with linear trend in demand and shortages. In same year Bhunia and Maiti gave an inventory model of deteriorating items with lot-size dependent replenishment cost and a linear trend in demand. In 2000, Giri et al wrote a note on a lot sizing heuristic for deteriorating items with time-varying demands and shortages. In 2002, Skouri and Papachristos did a continuous review inventory model, with deteriorating items, time varying demand, linear replenishment cost, partially time-varying backlogging. In 2004, Ghosh and Chaudhary provided an order level inventory model for a deteriorating item with Weibull distribution deterioration, time-quadratic demand, and shortages, in the same year Sana and Chaudhary wrote on a volume flexible production policy for a deteriorating item with time dependent demand and shortages. In 2009, Chakraborty et al studied production lot sizing with process deterioration and machine breakdown under inspection schedule. In 2012 Goyal et al gave an EOQ Model for Deteriorating Items with Stock Dependent Demand and Effect of Learning. In 2013, Goyal et al gave an EPQ model with stock dependent demand and time varying deterioration with shortages under inflationary environment. In 2015, Goyal et al gave an inventory model for deteriorated items with exponential demand under trade credit. Again, in the same year Goyal et al gave an EOQ inventory model with stock and selling price dependent demand rate, partial backlogging, and variable ordering cost. In 2016, Goyal and Chauhan wrote an EOQ Model for Deteriorating Items with Selling Price Dependent Demand Rate with Learning Effect. In the same year, Goyal et al wrote on supply Chain Model with Ramp Type Demand Under Planning Horizon. In 2017, Goyal et al provide a Mathematical Inventory Model for Deteriorating Items with Stock and Selling Price Dependent Demand and Partial Backlogging. In the same year, Goyal et al gave an EOQ model with Stock Dependent Demand and Partial Backlogging Under Inflation. Goyal and Agrawal provided a Production Inventory Model for Deteriorating Items with Price Dependent Demand Incorporated with Partially Backlogged Shortages in 2018. In same year Goyal et al wrote a Production Inventory Model with Selling Price and Stock Sensitive Demand and Partial Backlogging. In 2020, Singh et al did a relative study of crisp and fuzzy optimal reordering policy for perishable items. Kumari et al in 2022 gave an optimal Inventory Policy with Price-Dependent Demand and Variable Deterioration Rate also Delt with Trade Credit. In 2024, Goyal gave a replenishment policy for deteriorating items with order size dependent replenishment cost and time dependent demand.

In 2025, Patel et. al wrote on time-pproportional non-iinstantaneous ddeterioration decisions for vendor managed inventory system. This research paper initiated with an introductory part followed by the background of past study, the notation and assumptions use throughout this study, the limitations of the models are implied through these assumptions. After all this we described mathematical model without shortage in order to minimize the total cost in the planning horizon and establishes an optimal solution procedure under crisp environment then provides the solution in a fuzzy situation and shown the optimal solution procedure. Also, we provided numerical example to demonstrate the model also provided the special case of undiscounted. This model is developed for uncertain business houses where businessman is totally dependent on arrival of order and customer both, hence we provided a solution in this paper for those type of business houses.

Assumptions and Notations

i. The replenishment rate considers as infinite.
ii. The lead time is zero.
iii. A single item considered over a prescribed period of H units of time.
iv. The product demand rate, α units per year, is constant during the planning period.
v. The production rate is higher than the sum of consumption and deterioration rates.
vi. The time-varying decaying rate is denoted by $\theta(t) = \theta + bt$ where $0 \le b < 1, \quad \theta < 1$.
vii. The number of replenishment periods during the time horizon H is m
viii. Shortages are not allowed.
ix. C the unit cost of items, C_1 the inventory holding cost per unit per unit time and A the ordering cost per order.
x. C_R, total replenishment costs; C_P, total purchasing costs; C_H, total holding costs.
xi. Q, an optimal order quantity.
xii. R, representing the discount rate net of inflation.
xiii. T_j, the total time that elapsed up to and including the j^{th} replenishment cycle (j=1, 2,….., M), where T_m =H and T_0=0.
xiv. t_j, the time at which the inventory level in the j^{th} replenishment cycle drops to zero (j=1, 2, …, m).

2. Graded Mean Integration Representation Method

The graded mean integration representation method based on the integral value of graded mean h-level of a fuzzy number. Let $\tilde{\alpha} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ be a trapezoidal fuzzy number shown in figure 1, with membership function $\mu_{\tilde{\alpha}}(x)$, defined as

$$\mu_{\tilde{\alpha}}(x)=\begin{cases}0 & for -\infty < x < \alpha_1\\ \dfrac{x-\alpha_1}{a_2-\alpha_1} & for\, \alpha_1 \le x < \alpha_2\\ 1 & for\, \alpha_2 \le x < \alpha_3\\ \dfrac{\alpha_4-x}{\alpha_4-\alpha_3} & for\, \alpha_3 \le x < \alpha_4\\ 0 & for\, \alpha_4 < x < \infty\end{cases}$$

$$L_\alpha(x)=\frac{x-\alpha_1}{\alpha_2-\alpha_1},\ \alpha_1 \le x < \alpha_2$$
$$L^{-1}{}_\alpha(h)=\alpha_1+h\left(a_2-a_1\right), 0 \le h \le 1$$
$$R_\alpha(x)=\frac{\alpha_4-x}{\alpha_4-\alpha_3},\ \alpha_3 \le x < \alpha_4$$
$$R^{-1}{}_\alpha(h)=\alpha_4-h\left(\alpha_4-\alpha_3\right), 0 \le h \le 1$$

and $\mu_{\tilde{\alpha}}(x)$ satisfies the following conditions.

(1) $\mu_{\tilde{\alpha}}$ is a continuous mapping from R to the closed interval [0, 1],

(2) $\mu_\alpha = 0, -\infty < x \le \alpha_1,$

(3) μ_α = L(x) is strictly increasing on $[\alpha_1, \alpha_2]$,

(4) $\mu_\alpha = 1, \quad \alpha_2 \le x \le \alpha_3,$

(5) μ_α = R(x) is strictly decreasing on $[\alpha_3, \alpha_4]$,

(6) $\mu_\alpha = 0, \quad \alpha_4 \le x < \infty,$

where $\alpha_1, \alpha_2, \alpha_3\ and\ \alpha_4$ are real numbers and $\alpha_1 \le \alpha_2 \le \alpha_3 \le \alpha_4$.

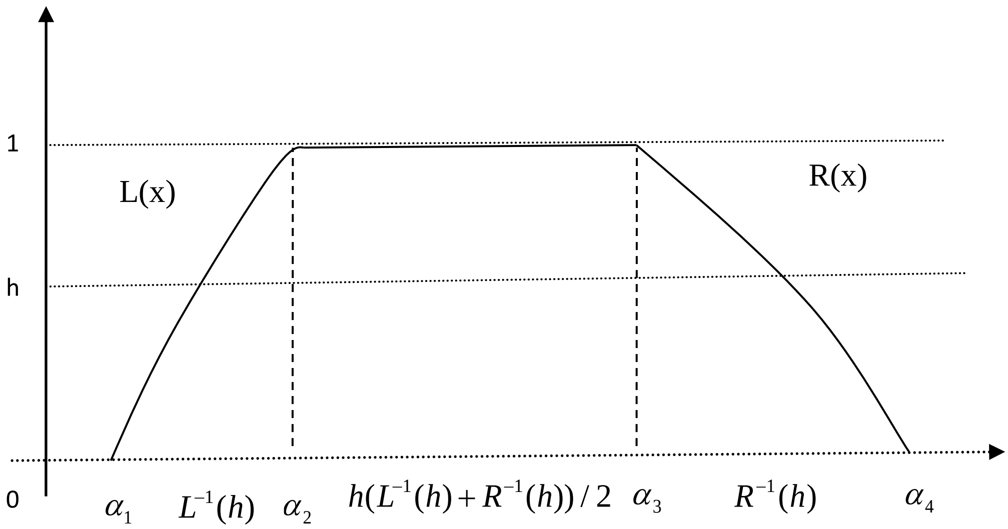

Figure 1. The graded mean h-level value of generalized fuzzy number

Also, this type of fuzzy number is denoted as $\tilde{\alpha}=(\alpha_1,\alpha_2,\alpha_3,\alpha_4)$.

By Graded Mean Integration Representation method L^{-1} and R^{-1} is an inverse function of L and R respectively, and the graded mean h-level value of a fuzzy number $\tilde{\alpha}=(\alpha_1,\alpha_2,\alpha_3,\alpha_4)$ is $h(L^{-1}(h)+R^{-1}(h))/2$ as figure 1. Then the graded mean integration representation of $\tilde{\alpha}\ is\, P(\tilde{\alpha})$ where

$$P(\tilde{\alpha}) = \int_0^1 h\left(\frac{L^{-1}(h)+R^{-1}(h)}{2}\right)dh \Big/ \int_0^1 hdh,$$

with $0 < h \leq 1$.

3. The Fuzzy Arithmetical Operations under Function Principal

Here we describe some fuzzy arithmetic operations under Function Principal as follows.

Suppose $\tilde{\alpha} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ and $\tilde{\beta} = (\beta_1, \beta_2, \beta_3, \beta_4)$ are two trapezoidal fuzzy numbers. Then,

(1) The addition of $\tilde{\alpha}\ and\ \tilde{\beta}$ is $\tilde{\alpha} \oplus \tilde{\beta} = (\alpha_1+\beta_1, \alpha_2+\beta_2, \alpha_3+\beta_3, \alpha_4+\beta_4)$, where $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3\ and\ \beta_4$ are any real numbers.

(2) The multiplication of $\tilde{\alpha}\ and\ \tilde{\beta}$ is $\tilde{\alpha} \otimes \tilde{\beta} = (c_1, c_2, c_3, c_4)$, where $P = \{\alpha_1\beta_1, \alpha_1\beta_4, \alpha_4\beta_1, \alpha_4\beta_4\}$, $q = \{a_2\beta_2, a_2\beta_3, a_3\beta_2, a_3\beta_3\}$, $c_1 = \min P, c_2 = \min q, c_3 = \max q\ \ and\ c_4 = \max P$.

(3) If $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3\ and\ \beta_4$ are all nonzero positive real numbers, then $\tilde{\alpha} \otimes \tilde{\beta} = (\alpha_1\beta_1, \alpha_2\beta_2, \alpha_3\beta_3, \alpha_4\beta_4)$.

(4) $-\tilde{\beta} = (-\beta_4, -\beta_3, -\beta_2, -\beta_1)$, Subtraction of $\tilde{\alpha}$ and $\tilde{\beta}$ is $\tilde{\alpha} - \tilde{\beta} = (\alpha_1-\beta_4, \alpha_2-\beta_3, \alpha_3-\beta_2, \alpha_4-\beta_1)$, where $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3\ and\ \beta_4$ are any real numbers.

(5) $1/\tilde{\beta} = (1/\beta_4, 1/\beta_3, 1/\beta_2, 1/\beta_1)$, Where $\beta_1, \beta_2, \beta_3\ and\ \beta_4$ are all nonzero positive real numbers?.

(6) If $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3\ and\ \beta_4$ are all nonzero positive real numbers, then the division of $\tilde{\alpha}\ and\ \tilde{\beta}$ is $\tilde{\alpha} \oslash \tilde{\beta} = (\alpha_1/\beta_4, \alpha_2/\beta_3, \alpha_3/\beta_2, \alpha_4/\beta_1)$.

4. Development of Mathematical Inventory Model

We develop and presents a system deteriorating products varying in time and establish optimum refilling plan related to cycle duration during the time horizon *H*, lie in negative and positive inventories duration for without shortfalls to get specific results. Furthermore, we establish that total multi-variant price exercises are bulging outward. With respect to the factors of the system carried out, numerical illustration is helpful to present the pertinence of the proposed models and analysis of sensitivity.

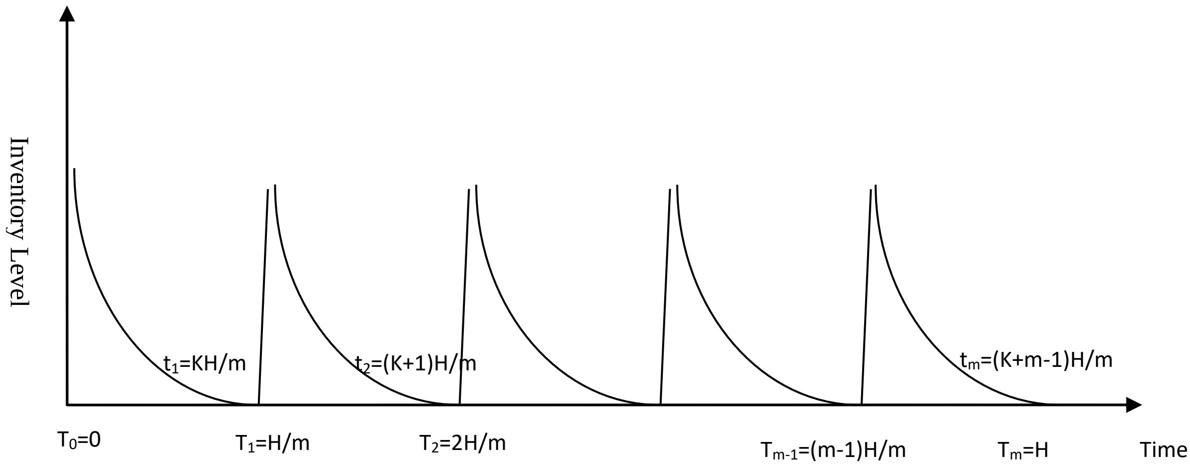

Figure 2. Pictorial representation of the inventory cycles without Shortage

4.1 Under Crisp Set Terminology

A realization of the inventory level in the system is given in Fig 1. For the development of the model, we assume that there are m cycles during the real-time horizon H each of length T so that T=H/m. Hence, T_j = j T (j= 0,1, 2,…..,m) are the reorder times over the planning horizon H . Initially, the inventory level I(t) during the first replenishment cycle. This inventory level is depleted by the effects of demand and deterioration rate. So, the variation of I(t) with respect to t is governed by the following differential equation:

$$\frac{dI(t)}{dt} = -\alpha - \left(\theta + bt\right) I(t)\,; \qquad 0 \le t \le T \tag{1}$$

with boundary condition I(T) = 0.

The simplification of (1) can be represented by

$$I\left(t\right) = \alpha\left[\left(T-t\right) + \frac{\theta}{2}\left(T^2 - t^2\right) + \frac{\left(b+\theta^2\right)}{6}\left(T^3 - t^3\right)\right] \times \left[1 - \left\{\theta t + \left(b-\theta^2\right)\frac{t^2}{2}\right\}\right]\,; \quad 0 \le t \le T\,. \tag{2}$$

Since there are m replenishments in the entire time horizon H,

The present values of the total replenishment costs are given by

$$C_R = A\sum_{j=0}^{m-1} e^{-RT_j} = A\frac{\left(1-e^{-RH}\right)}{\left(1-e^{\frac{-RH}{m}}\right)}. \tag{3}$$

The present values of total purchasing costs are

$$C_P = C\sum_{j=1}^{m} I(0)e^{-RT_{j-1}} = \alpha C\sum_{j=1}^{m}\left\{T + \frac{\theta T^2}{2} + \left(b+\theta^2\right)\frac{T^3}{6}\right\}\{1-0\}\,e^{-T_{j-1}}$$

$$C_P = \alpha C\left[T + \frac{\theta T^2}{2} + \left(b+\theta^2\right)\frac{T^3}{6}\right]\frac{\left(1-e^{-RH}\right)}{\left(1-e^{-RH/m}\right)} \tag{4}$$

The present values of the holding costs during the first replenishment cycle are

$$H_1 = C_1\int_0^T I\left(t\right)e^{-Rt}\,dt$$

$$H_1 = \alpha C_1\left[\left\{T+\frac{\theta T^2}{2}+\frac{\left(b+\theta^2\right)}{6}T^3\right\}\left(\frac{e^{-Rt}}{-R}+\frac{1}{R}\right)+\left\{1+\theta T+\frac{\theta^2 T^2}{2}+\frac{\left(b+\theta^2\right)\theta T^3}{6}\right\}\left\{\frac{Te^{-RT}}{R}+\frac{e^{-RT}}{R^2}-\frac{1}{R^2}\right\}\right] \tag{5}$$

Hence, the present values of the total holding costs during the entire time horizon *H* are given as

$$C_H = \sum_{j=1}^{m} H_1 e^{-RT_{j-1}} = \alpha C_1\left[\left\{\frac{H}{m}+\frac{\theta H^2}{2m^2}+\frac{\left(b+\theta^2\right)H^3}{6m^3}\right\}\left(\frac{e^{-RH/m}}{-R}+\frac{1}{R}\right)\right.$$
$$\left.+\left\{1+\frac{\theta H}{m}+\frac{\theta^2 H^2}{2m^2}+\frac{\left(b+\theta^2\right)\theta H^3}{6m^3}\right\}\left\{\frac{He^{-RT}}{Rm}+\frac{e^{-RH/m}}{R^2}-\frac{1}{R^2}\right\}\right]\left(\frac{1-e^{-RH}}{1-e^{-RH/m}}\right) \tag{6}$$

The net present total variable cost TC of the system during the entire time Horizon *H* is the sum of the replenishment cost C_R, the purchasing cost C_p and holding cost C_H

$$TC\left(m\right) = C_R + C_P + C_H \tag{7}$$

A mathematical model is derived to obtain for the optimal replenishment when Total cost is minimized. Minimize TC(m) ; such that m > 0.

4.2 Under Fuzzy Set Terminology

A realization of the inventory level in the Fuzzy system as described in fuzzy in the crisp model. For the development of the fuzzy model, we consider the same assumption in a fuzzy environment that there are m cycles during the fuzzy time horizon each of length T so that $\mathrm{T} = \tilde{\mathrm{H}}/\mathrm{m}$. Hence, the reorder times over the planning horizon $\tilde{\mathrm{H}}$ are T_j = jT (*j*= 0,1,2,…..,*m*). Initially, consider the Fuzzy inventory level $\tilde{I}(t)$ during the first replenishment cycle. This inventory level is depleted by the effects of demand and deterioration rate. So, the variation of $\tilde{I}(t)$ with respect to *t* is governed by the following differential equation:

$$\frac{d\,\tilde{I}(t)}{dt} = -\tilde{\alpha} - \left(\tilde{\theta} + \tilde{b}\,t\right)\tilde{I}(t) \qquad 0 \le t \le T \tag{8}$$

With the boundary condition I(T) = 0.The solution of (1) can be represented by

$$\tilde{I}(t) = \tilde{\alpha}\left[\left(T-t\right)+\frac{\tilde{\theta}}{2}\left(T^2-t^2\right)+\frac{\left(\tilde{b}+\tilde{\theta}^2\right)}{6}\left(T^3-t^3\right)\right]\times\left[1-\left\{\tilde{\theta}t+\left(\tilde{b}-\tilde{\theta}^2\right)\frac{t^2}{2}\right\}\right] \qquad 0 \le t \le T \tag{9}$$

Since there are *m* replenishments in the entire time horizon *H*,
The present values of the total replenishment costs are given by

$$\tilde{C}_R = \tilde{A}\sum_{j=0}^{m-1} e^{-\tilde{R}\,T_j} = \tilde{A}\frac{\left(1-e^{-\tilde{R}\tilde{H}}\right)}{\left(1-e^{-\tilde{R}\tilde{H}/m}\right)} \tag{10}$$

The present values of total purchasing costs are

$$\tilde{C}_p = \tilde{C}\sum_{j=1}^{m}\tilde{I}(0)e^{-\tilde{R}T_{j-1}} = \tilde{\alpha}\,\tilde{C}\left[T+\frac{\tilde{\theta}\,T^2}{2}+\left(b+\tilde{\theta}^2\right)\frac{T^3}{6}\right]\frac{\left(1-e^{-\tilde{R}\tilde{H}}\right)}{\left(1-e^{-\frac{\tilde{R}\tilde{H}}{m}}\right)} \tag{11}$$

The present values of the holding costs during the first replenishment cycle are

$$\tilde{H}_1 = \tilde{C}_1\int_0^T I\left(t\right)e^{-\tilde{R}\,t}dt$$

$$\tilde{H}_1 = \tilde{\alpha}\,\tilde{C}_1\left[\left\{T + \frac{\tilde{\theta}\,T^2}{2} + \frac{\left(\tilde{b}+\tilde{\theta}^2\right)}{6}T^3\right\}\left(\frac{e^{-\tilde{R}\,t}}{-\tilde{R}} + \frac{1}{\tilde{R}}\right) + \left\{1+\tilde{\theta}\,T + \frac{\tilde{\theta}^2T^2}{2} + \frac{\left(\tilde{b}+\tilde{\theta}^2\right)\tilde{\theta}T^3}{6}\right\}\left\{\frac{Te^{-\tilde{R}\,T}}{\tilde{R}} + \frac{e^{-\tilde{R}\,T}}{\tilde{R}^2} - \frac{1}{\tilde{R}^2}\right\}\right] \tag{12}$$

Hence, the present values of the total holding costs during the entire time horizon H are given as

$$\tilde{C}_H = \sum_{j=1}^{m}\tilde{H}_1 e^{-\tilde{R}\,T_{j-1}} = \tilde{\alpha}\tilde{C}_1\left[\left\{\frac{\tilde{H}}{m} + \frac{\tilde{\theta}\tilde{H}^2}{2m^2} + \frac{\left(\tilde{b}+\tilde{\theta}^2\right)\tilde{H}^3}{6m^3}\right\}\left(\frac{e^{-\tilde{R}\tilde{H}/m}}{-\tilde{R}} + \frac{1}{\tilde{R}}\right)\right.$$
$$\left. + \left\{1 + \frac{\tilde{\theta}\tilde{H}}{m} + \frac{\tilde{\theta}^2\tilde{H}^2}{2m^2} + \frac{\left(\tilde{b}+\tilde{\theta}^2\right)\tilde{\theta}\tilde{H}^3}{6m^3}\right\}\left\{\frac{\tilde{H}e^{-\tilde{R}T}}{\tilde{R}m} + \frac{e^{-\tilde{R}\tilde{H}/m}}{\tilde{R}^2} - \frac{1}{\tilde{R}^2}\right\}\right]\left(\frac{1-e^{-\tilde{R}\tilde{H}}}{1-e^{-\tilde{R}\tilde{H}/m}}\right) \tag{13}$$

The net present total variable cost TC of the system during the entire time Horizon H is the sum of the replenishment cost, the purchasing cost $\tilde{C}_p$ and holding the cost $\tilde{C}_H$

$$T\tilde{C}(m) = \tilde{C}_R + \tilde{C}_p + \tilde{C}_H \tag{14}$$

A mathematical model is derived to obtain the optimal replenishment when $T\tilde{C}(m)$ is minimized.

Minimize: $T\tilde{C}(m)$ such that m > 0.

4.3 Optimal Solution Procedure in the Fuzzy Set Terminology

Our aim is to find out the optimal value of m that minimize the total system cost $T\tilde{C}(m)$, here optimization technique is used to minimize m as follow

Step1. Since m is the number of the cycle in the planning horizon, consider as an integer value, start by choosing an integer value of m ≥1.

Step2: For, m=1, take the derivative of $T\tilde{C}(m)$ with respect to m and equate the results to zero, the necessary conditions for optimality are $dT\tilde{C}(m)/dm$=0 this equation resolve for m .

Step3: Using m found in step 2, substitute into equation (7) and derive $T\tilde{C}(m)$

Step4: Repeat step 2 and 3 for all possible m values until the minimum $T\tilde{C}(m^*)$ is found. The $T\tilde{C}(m^*)$ value constitutes the optimal solution that satisfied the following conditions:

$d^2T\tilde{C}(m^*)/dm^2 > 0$ for m* and $T\tilde{C}(m+1) \geq T\tilde{C}(m)$

Step5: For optimal value m*, we can find the optimal-order quantity is $Q = \tilde{\alpha}\left[\left(\frac{\tilde{H}}{m}\right) + \frac{\tilde{\theta}}{2}\left(\frac{\tilde{H}^2}{m^2}\right) + \frac{\left(\tilde{b}+\tilde{\theta}^2\right)}{6}\left(\frac{\tilde{H}^3}{m^3}\right)\right]$ by using Equation (9).

Numerical example (Crisp Set Terminology) To illustrate the effect of the general model developed in this paper with the following data:

The demand parametric value is α=600unit/year, the deterioration rate of the on-hand inventory per unit time is θ=.20, A= $ 120.00, C_1=$1.75 per unit per year, C=$2 per unit, R=0.50, b=0.05, the time horizon H=10 yr.

By using the solution procedure described in the section 5, the computational result are shown in Table 1, From this table, we analysis that that the optimal replenishment number m*=11, the total variable cost TC become minimum. During the first replenishment cycle order quantity

Q*=601.803and minimum total variable cost TC*(m) = 16284.7, We then have the time interval between replenishment is T=(H/m) = 10/11=0.901 year

Numerical Example (Fuzzy Set Terminology) Similarly, To illustrate the effect of the general model developed in this paper with the following data under fuzzy environment:

The fuzzy demand parametric value is $\tilde{\alpha} = (480, 540, 660, 720)$ the unit/year, the deterioration rate of the on-hand inventory per unit time is $\theta = (0.16, 0.18, 0.22, 0.24)$, $\tilde{C}_1 = (1.40, 1.575, 1.925, 2.10)$ per unit per year, $\tilde{c} = (1.6, 1.8, 2.2, 2.4)$ per unit, $\tilde{b} = (0.40, 0.45, 0.55, 0.60)$, the time horizon $\tilde{H} = (9, 10, 11, 12)$ yr.

By using the solution procedure described in the section 5, the computational result is Shown in Table 1, From this table, we see that that the optimal replenishment number **m*=9**, the total variable cost TC become minimum. During the first replenishment cycle order quantity Q*=**819** and minimum total variable cost TC*(m) = **11601**, we then have the time interval between replenishment is $\tilde{T} = \tilde{H} / m = (\frac{9}{9}, \frac{10}{9}, \frac{11}{9}, \frac{12}{9})$ the year

Table 1						
Crisp Models				Fuzzy Models		
M	TC(m)	Q		m	TC(m)	Q
1	149172	21000		1	111254	26283
2	45889	5625		2	31922	6600
3	28993	3000		3	19794	3412
4	22829	2016		4	15517	2254
5	19851	1512		5	13519	1674
6	18218	1208		6	12482	1329
7	17274	1006		7	11938	1101
8	16728	861		8	11679	939
9	16430	753		**9**	**11601**	**819**
10	16300	669		10	11642	726
11	**16285**	**602**		11	11767	652
12	16354	547		12	11954	592
13	16487	501		13	12186	542
14	16668	462		14	12454	499
15	16887	429		15	12749	463
16	17136	401		16	13066	432
17	17410	376		17	13401	405
18	17704	353		18	13750	381
19	18014	334		19	14112	359
20	18339	316		20	14483	340

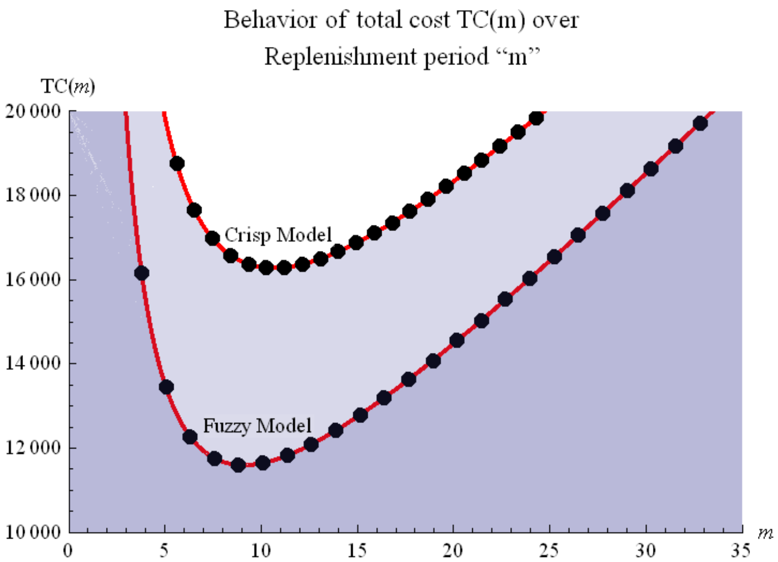

Figure 3: Change in T.C. over m

Special case: In this section, we explain the significant case R=0, and compare the differences Crisp Model and fuzzy Model with classical EOQ model.

(Undiscounted in Crisp Model)

When R=0, then the total (undiscounted) variable cost, TC (T) is

$$C_h = \alpha C_1 \left(T^2 + \frac{T^3\theta}{2} + \frac{T^4(b+\theta^2)}{6} \right) \frac{H}{T} \left(\left(\frac{H}{T} + 1 \right) \Big/ 2 \right);$$

$$C_R = A \left(\frac{H}{T} \right) \left(\frac{H}{T} - 1 \right) \Big/ 2\,;$$

$$C_p = \left(\alpha\, C\, \left(H/T \right) \left(H/T + 1 \right) \big/ 2 \right) \left[T + \frac{\theta\, T^2}{2} + (b+\theta^2) \frac{T^3}{6} \right].$$

$$\text{TC(T)} = C_R + C_p + C_H$$

The necessary condition for minimum total variable cost is $\frac{dTC(T)}{dT} = 0$, giving

$$-\frac{AH^2}{2T^3} - \frac{AH(\frac{H}{T}-1)}{2T^2} - \frac{H^2\{\frac{1}{6}CT^3\alpha(b+\theta^2)) + \frac{1}{2}CT^2\alpha\theta + CT\alpha\}}{2T^3}$$
$$-\frac{C_1 H^2 \alpha\{\frac{1}{6}T^3(b+\theta^2) + \frac{T^2\theta}{2} + T\}}{2T^2} + \frac{H\left(\frac{H}{T}+1\right)\left(\frac{1}{2}CT^2\alpha(b+\theta^2) + CT\alpha\theta + C\alpha\right)}{2T} + \frac{1}{2}C_1 H\alpha \left(\frac{H}{T}+1\right)\left(\frac{1}{2}T^2(b+\theta^2) + T\theta + 1\right)$$
$$-\frac{H\left(\frac{H}{T}+1\right)\left(\frac{1}{6}CT^3\alpha(b+\theta^2) + \frac{1}{2}CT^2\alpha\theta + CT\alpha\right)}{2T^2} = 0 \tag{15}$$

The optimum value of T can be obtained from expression (8) by using the Cost minimization technique. Taking the optimum value of T, we can also obtain the optimum-order quantity by Eq. (2) is

$$Q = [T + \frac{\theta \mathrm{T}^2}{2} + (b+\theta^2)\frac{T^3}{6}]\alpha \ . \tag{16}$$

When R=0 and H=1.0, with the same values of different parameters as taken in the numerical example of Section 3.3, Equations (7) to (9) we get the optimal replenishment cycle length, T, order quantity, Q, replenishment cost, CR, purchasing cost, CP, holding cost, CH and total system cost, TC as follow:

$T^* = 0.919015$; $TC^* = 2502.48$; Q^*=609.07; C_R^*=5.75325; $C_P^* = 1383.89$; $C_H^* = 1112.84$

(Undiscounted in Fuzzy Model) Now, we find that the optimal-order quantity and all the relative system costs under Fuzzy as using the same parameter value are as follow:

$T\tilde{C} = 1701.99$; $\tilde{Q} = 286.85$; $\tilde{C}_R = 250.99$; $\tilde{C}_P = 1200$; $\tilde{C}_H = 250.99$

When we compare the values among Q*, C_R*, C_P*, C_H* TC* and $\tilde{Q}, \tilde{C}_R, \tilde{C}_P, \tilde{C}_H, T\tilde{C}$

we notice that $Q^* > \tilde{Q}$, $C_R^* < \tilde{C}_R$, $C_P^* > \tilde{C}_P$, $C_H^* > \tilde{C}_H$, $TC^* > T\tilde{C}$

This observable fact is due to the deterioration rate. So that when an inventory system included deterioration, the system will increase order quantity to avoid Shortage but accompany higher purchasing and holding cost, So the total system cost is lesser than that of the crisp model.

Conclusion

This model incorporates some realistic facts that are likely to be associated with the inventory of some kinds of goods. Deterioration of items over time is a natural feature for goods. The DCF approach permits a proper recognition of the financial implication of the opportunity cost in inventory analysis. In keeping with this reality, these factors are incorporated into the present models under fuzzy. We have given an analytic formulation of the problem on the fuzzy framework described above and have presented an optimal solution procedure to find optimal inventory replenishment policies in a fuzzy system. Finally, it is seen that if we consider consumption rate parameter (α), the unit cost (C) and unit shortage cost (C_2) under fuzzy environment has highly significant effects on the order quantity. The total system cost is also reduced under fuzzy environment corresponding to the consumption rate parameter (α), the unit cost (C) and discount rate parameter (R). This model is developed for uncertain business houses where businessman is totally dependent on arrival of order and customer both, hence we provided a solution in this paper for those type of business houses.

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