Received
December 22, 2025

Revised
January 16, 2026

Accepted
January 25, 2026

Published
February 03, 2026

Keywords
HyperStructure, Super-Hyperstructure, Network topology, Network hypertopology, Network super-hypertopology.

A Proposal for Network Hypertopology and Super-hypertopology: A Framework for Multi-Level Network Structures

Takaaki Fujita †

† Independent Researcher, Tokyo, Japan.

Corresponding Author: takaaki.fujita060@gmail.com

ORCID: https://orcid.org/0009-0007-1509-2728

DOI/url: https://journalmanager.transitus.in/index.php/jamss

Abstract

We introduce the notions of Network Hypertopology and Network Super-hypertopology, which extend the classical graph-based model of network topology to higher-order structures on its power sets. A network topology $G = (V, E)$ encodes connectivity, directionality, and link metrics among devices. By endowing the hyperspace $\mathcal{P}(V)$ with the Vietoris (hypertopology) topology, we lift these closure axioms to families of node-sets. Iterating this construction across iterated power sets $\mathcal{P}^n(V)$ yields a Super-hypertopology that maintains arbitrary-union and finite-intersection closure at every level. While we establish the formal definitions and foundational properties of these higher-order topologies, their practical applications and empirical evaluation remain open for future investigation.

1 Preliminaries

In this section, we establish notation and recall basic concepts that will be used throughout. We assume familiarity with elementary set theory and topology; for further details, see the cited references. The concepts discussed in this paper are assumed to be finite.

1.1 Hyperstructures and Their Iterations

The notion of a *hyperstructure* arises by replacing an underlying set with its power set, thereby allowing operations on collections of elements rather than on individual elements [1, 2]. Iterating this construction leads to *superhyperstructures*, which capture multi-level, hierarchical relationships [3, 4].

Definition 1.1 (Base Set)**.** Let S be a nonempty set, called the *base set*. All subsequent constructions—subsets, power sets, and their iterates—are formed from S.

Definition 1.2 (Power Set)**.** For any set S, its *power set* is

$$\mathcal{P}(S) = \{ A \mid A \subseteq S \},$$

the collection of all subsets of S, including both the empty set and S itself.

Definition 1.3 (n^{th} Power Set)**.** [5, 6] Let H be a set. We define the *iterated power sets*

$$\mathcal{P}^1(H) = \mathcal{P}(H), \qquad \mathcal{P}^{k+1}(H) = \mathcal{P}(\mathcal{P}^k(H)) \quad (k \geq 1).$$

That is, $\mathcal{P}^n(H)$ is the result of applying the power-set operator n times to H.

If one wishes to exclude the empty set at each stage, one can similarly define the *reduced iterates*

$$\mathcal{P}_1^*(H) = \mathcal{P}(H) \setminus \{\emptyset\}, \qquad \mathcal{P}_{k+1}^*(H) = \mathcal{P}^*(\mathcal{P}_k^*(H)).$$

Example 1.4 (Iterated Power Sets in a Hierarchical Sensor Network)**.** Let $H = \{S_1, S_2, S_3\}$ be the set of three wireless sensors deployed in an Internet-of-Things application. Then:

$$\mathcal{P}^1(H) = \mathcal{P}(H) = \{ \emptyset, \{S_1\}, \{S_2\}, \{S_3\}, \{S_1, S_2\}, \{S_1, S_3\}, \{S_2, S_3\}, \{S_1, S_2, S_3\}\}.$$

Each nonempty subset in $\mathcal{P}^1(H)$ represents a possible sensor-group for local data aggregation or collaborative sensing.

Next, the second-level power set

$$\mathcal{P}^2(H) = \mathcal{P}(\mathcal{P}(H))$$

consists of all collections of sensor-groups. For instance, the element

$$= \{\{S_1, S_2\}, \{S_3\}\} \in \mathcal{P}^2(H)$$

encodes a two-cluster configuration in which S_1 and S_2 form one cluster and S_3 stands alone.

At the third level,

$$\mathcal{P}^3(H) = \mathcal{P}(\mathcal{P}^2(H)),$$

each element is a collection of clusterings. For example,

$$\Gamma = \Big\{ \{\{S_1, S_2\}, \{S_3\}\}, \{\{S_1\}, \{S_2, S_3\}\} \Big\} \in \mathcal{P}^3(H)$$

represents a choice between two different clustering schemes at the meta-level.

In general, $\mathcal{P}^n(H)$ captures n-tier hierarchies of sensor groupings, enabling multi-scale modeling of network structure and control.

Definition 1.5 (Classical Structure)**.** (cf. [7, 8]) A *Classical Structure* is a mathematical framework defined on a non-empty set H, characterized by one or more *Classical Operations* that adhere to specific *Classical Axioms*. Formally:

A *Classical Operation* is a function of the form:

$$\#_0 : H^m \to H,$$

where $m \geq 1$ denotes a positive integer, and H^m represents the m-fold Cartesian product of H. Examples include algebraic operations such as addition and multiplication in structures like groups, rings, and fields.

Definition 1.6 (Hyperstructure)**.** (cf. [7]) A *Hyperstructure* extends the concept of a Classical Structure by operating on the powerset of a base set. It is formally defined as:

$$\mathcal{H} = (\mathcal{P}(S), \circ),$$

where S is the base set, $\mathcal{P}(S)$ denotes its powerset, and $\circ$ is an operation defined for subsets within $\mathcal{P}(S)$.

Example 1.7 (Hyperstructure of a Modular Drone Assembly)**.** In a simple modular drone design one considers the following set of basic components:

$$S = \{\text{Frame}, \ \text{Motor}, \ \text{Propeller}, \ \text{Battery}\}.$$

Its power set

$$\mathcal{P}(S) = \{\varnothing, \{\text{Frame}\}, \{\text{Motor}\}, \{\text{Propeller}\}, \{\text{Battery}\}, \{\text{Frame,Motor}\}, \dots, \{\text{Frame,Motor,Propeller,Battery}\}\}$$

consists of all possible (partial or complete) assemblies of these components. We define a binary hyperoperation

$$\circ : \mathcal{P}(S) \times \mathcal{P}(S) \longrightarrow \mathcal{P}(S), \qquad A \circ B = A \cup B,$$

which merges two subassemblies into their union. For example,

$$\{\text{Frame,Motor}\} \circ \{\text{Propeller,Battery}\} = \{\text{Frame,Motor,Propeller,Battery}\},$$

yields the full drone configuration. The pair $(\mathcal{P}(S), \circ)$ thus forms a *hyperstructure* encoding the space of all modular drone assemblies, where "adding" two configurations nondeterministically produces any combined assembly in the union of their parts.

Definition 1.8 (n^{th} Superhyperstructure)**.** [4,9] Let S be a nonempty set (the *base set*). Define iteratively for $k \geq 0$:

$$\mathcal{P}_0(S) = S, \qquad \mathcal{P}_{k+1}(S) = \mathcal{P}(\mathcal{P}_k(S)),$$

where $\mathcal{P}$ denotes the ordinary power–set. Fix an integer $n \geq 1$. The n^{th} *Superhyperstructure* on S is the pair

$$\mathcal{SH}_n = (\mathcal{P}_n(S), \circ_n),$$

where $\circ_n$ is the binary operation

$$\circ_n : \mathcal{P}_n(S) \times \mathcal{P}_n(S) \longrightarrow \mathcal{P}_n(S), \qquad A \circ_n B = A \cup B.$$

Example 1.9 (2-Superhyperstructure of a Modular Drone)**.** Let the base set of drone components be

$$S = \{\text{Frame}, \ \text{Motor}, \ \text{Propeller}, \ \text{Battery}\}.$$

Then

$$\mathcal{P}_1(S) = \mathcal{P}(S)$$

is the set of all *subassemblies*, such as {Frame,Motor} or {Propeller,Battery}. Next,

$$\mathcal{P}_2(S) = \mathcal{P}(\mathcal{P}(S))$$

consists of all *collections of subassemblies*. For instance, define two level-2 elements:

$$A = \{\{\text{Frame,Motor}\}, \ \{\text{Frame,Propeller}\}\}, \quad B = \{\{\text{Battery}\}, \ \{\text{Propeller,Battery}\}\}.$$

Here A groups the structural and propulsion subassemblies, while B gathers the power units. The binary operation on $\mathcal{P}_2(S)$ is

$$A \circ_2 B = A \cup B = \{\{\text{Frame,Motor}\}, \{\text{Frame,Propeller}\}, \{\text{Battery}\}, \{\text{Propeller,Battery}\}\},$$

which merges two collections into the full set of all four subassembly types. Thus $(\mathcal{P}_2(S), \circ_2)$ is a *2-Superhyperstructure* encoding both first-order subassemblies and their groupings in a single unified framework.

1.2 Topology, Hypertopology, and Super-hypertopology

We begin with the familiar concept of a topology on a set and then show how it may be lifted first to the power-set and subsequently to iterated power-sets, yielding the notions of hypertopology and Super-hypertopology, respectively.

Definition 1.10 (Topology)**.** (cf. [10, 11]) Let X be a nonempty set. A *topology* on X is a collection

$$\tau \subseteq \mathcal{P}(X)$$

such that:

1. $\varnothing \in \tau$ and $X \in \tau$.
2. $\bigcup_{\alpha \in I} U_\alpha \in \tau$ for any family $\{U_\alpha\}_{\alpha \in I} \subseteq \tau$.
3. $\bigcap_{k=1}^{n} U_k \in \tau$ for every finite subcollection $U_1, \ldots, U_n \in \tau$.

The pair (X, τ) is called a *topological space*.

Definition 1.11 (Hypertopology)**.** (cf. [12–14]) Let H be a nonempty set and write $\mathcal{P}(H)$ for its power set. A *hypertopology* on $\mathcal{P}(H)$ is any topology

$$\tau_H \subseteq \mathcal{P}(\mathcal{P}(H))$$

satisfying:

1. $\varnothing \in \tau_H$ and $\mathcal{P}(H) \in \tau_H$.
2. $\bigcup_{\beta \in B} V_\beta \in \tau_H$ for any family $\{V_\beta\}_{\beta \in B} \subseteq \tau_H$.
3. $\bigcap_{j=1}^{m} V_j \in \tau_H$ for every finite subcollection $V_1, \ldots, V_m \in \tau_H$.

The resulting pair $(\mathcal{P}(H), \tau_H)$ is called a *hypertopological space*.

Example 1.12 (Vietoris Hypertopology on a Two-Element Set)**.** Let $H = \{a, b\}$. Then its power set is

$$\mathcal{P}(H) = \{\, \emptyset,\ \{a\},\ \{b\},\ \{a, b\}\}.$$

Equip H with the discrete topology $\tau_H^0 = \mathcal{P}(H)$. The *Vietoris hypertopology* τ_H on $\mathcal{P}(H)$ is the topology generated by the subbasis

$$\langle U \rangle = \{\, A \subseteq H \mid A \subseteq U\}, \quad [U] = \{\, A \subseteq H \mid A \cap U \neq \emptyset\},$$

for each $U \in \tau_H^0$. Concretely, the nontrivial subbasic opens are:

$$\langle \{a\} \rangle = \{\emptyset, \{a\}\}, \quad [\{a\}] = \{\{a\}, \{a, b\}\},$$

$$\langle \{b\} \rangle = \{\emptyset, \{b\}\}, \quad [\{b\}] = \{\{b\}, \{a, b\}\}.$$

A typical *basic open* set is a finite intersection of these subbasic opens. For instance,

$$[\{a\}] \cap [\{b\}] = \{\{a, b\}\},$$

so the singleton $\{\{a, b\}\}$ is open in τ_H. One checks easily that τ_H contains $\varnothing$ and $\mathcal{P}(H)$, is closed under arbitrary unions and finite intersections, and hence $(\mathcal{P}(H), \tau_H)$ is a bona fide hypertopological space.

Definition 1.13 (Super-hypertopology)**.** (cf. [12, 13]) Let H be a nonempty set, and define recursively

$$\mathcal{P}_0(H) = H, \qquad \mathcal{P}_{k+1}(H) = \mathcal{P}(\mathcal{P}_k(H)) \quad (k \geq 0).$$

Fix $n \geq 1$. A *Super-hypertopology* on $\mathcal{P}_n(H)$ is a topology $\tau_{SH} \subseteq \mathcal{P}(\mathcal{P}_n(H))$ satisfying:

1. $\emptyset \in \tau_{SH}$ and $\mathcal{P}_n(H) \in \tau_{SH}$.
2. $\bigcup_{\gamma \in \Gamma} W_\gamma \in \tau_{SH}$ for any family $\{W_\gamma\}_{\gamma \in \Gamma} \subseteq \tau_{SH}$.
3. $\bigcap_{i=1}^{p} W_i \in \tau_{SH}$ for any finite subcollection $W_1, \ldots, W_p \in \tau_{SH}$.

The pair $(\mathcal{P}_n(H), \tau_{SH})$ is called a *superhypertopological space*.

Example 1.14 (2-Super-hypertopology on a Two-Point Set)**.** Let $H = \{a, b\}$. As in Example 1.12, the first iterated hyperspace is

$$\mathcal{P}_1(H) = \mathcal{P}(H) = \{\varnothing, \{a\}, \{b\}, \{a, b\}\},$$

equipped with the Vietoris hypertopology $\tau^{(1)}$. The second iterated hyperspace is

$$\mathcal{P}_2(H) = \mathcal{P}(\mathcal{P}_1(H)),$$

the set of all subsets of $\mathcal{P}_1(H)$, which has $2^4 = 16$ elements.

We define the *2-Super-hypertopology* $\tau^{(2)}$ on $\mathcal{P}_2(H)$ to be the Vietoris topology generated by subbasic opens of the form

$$\langle U \rangle = \{\, A \subseteq \mathcal{P}_1(H) \mid A \subseteq U \}, \quad [U] = \{\, A \subseteq \mathcal{P}_1(H) \mid A \cap U \neq \varnothing \},$$

for each $U \in \tau^{(1)}$.

Concrete subbasic opens. For example, take

$$U = \langle \{a\} \rangle = \{\varnothing, \{a\}\} \;\in\; \tau^{(1)}.$$

Then at level 2:

$$\langle U \rangle = \{\, A \subseteq \mathcal{P}_1(H) \mid A \subseteq \{\varnothing, \{a\}\} \} = \{\, \emptyset,\; \{\varnothing\},\; \{\{a\}\},\; \{\varnothing, \{a\}\} \},$$

$$[U] = \{\, A \subseteq \mathcal{P}_1(H) \mid A \cap \{\varnothing, \{a\}\} \neq \emptyset \} = \mathcal{P}_2(H) \;\setminus\; \{\, A \mid A \subseteq \{\{b\}, \{a, b\}\} \}.$$

A basic open. A typical basic open in $\tau^{(2)}$ is a finite intersection of such subbasic opens. For instance,

$$\langle U \rangle \;\cap\; [\{\{b\}\}] = \{\, \{\{a\}\},\; \{\varnothing, \{a\}\} \},$$

since $\{\{b\}\} \in \tau^{(1)}$ and $[\{\{b\}\}] = \{\, A \subseteq \mathcal{P}_1(H) \mid \{b\} \in A \}$.

One verifies that $\tau^{(2)}$ contains $\varnothing$ and $\mathcal{P}_2(H)$, is closed under arbitrary unions and finite intersections, and thus $(\mathcal{P}_2(H), \tau^{(2)})$ is a valid *2-superhypertopological space*.

Example 1.15 (3-Super-hypertopology on a Two-Point Set)**.** Let $H = \{a, b\}$. Define iteratively

$$\mathcal{P}_1(H) = \mathcal{P}(H) = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}, \quad \mathcal{P}_2(H) = \mathcal{P}(\mathcal{P}_1(H)), \quad \mathcal{P}_3(H) = \mathcal{P}(\mathcal{P}_2(H)).$$

Equip $\mathcal{P}_1(H)$ with the Vietoris hypertopology $\tau^{(1)}$, and $\mathcal{P}_2(H)$ with the 2-Super-hypertopology $\tau^{(2)}$ generated by subbasic opens

$$\langle U\rangle = \{A \subseteq \mathcal{P}_1(H) \mid A \subseteq U\}, \quad [U] = \{A \subseteq \mathcal{P}_1(H) \mid A \cap U \neq \emptyset\},$$

for all $U \in \tau^{(1)}$. Concretely, set

$$U_1 \;=\; \langle\{a\}\rangle \;=\; \{\varnothing, \{a\}\} \;\in \tau^{(1)},$$

and define the 2-level basic open

$$U_2 \;=\; \langle U_1\rangle \;\cap\; [\{\{b\}\}] \;\subseteq\; \mathcal{P}_2(H).$$

Now $\mathcal{P}_3(H)$ carries the 3-Super-hypertopology $\tau^{(3)}$ generated by subbasis

$$\langle U_2\rangle = \{X \subseteq \mathcal{P}_2(H) \mid X \subseteq U_2\}, \qquad [U_2] = \{X \subseteq \mathcal{P}_2(H) \mid X \cap U_2 \neq \emptyset\},$$

together with those arising from any open in $\tau^{(2)}$. For instance, a typical basic open in $\tau^{(3)}$ is

$$\langle U_2\rangle \;\cap\; [\langle U_1\rangle] \;=\; \{X \subseteq \mathcal{P}_2(H) \;\mid\; X \subseteq U_2,\; X \cap \langle U_1\rangle \neq \emptyset\}.$$

One checks that $\tau^{(3)}$ contains $\varnothing$ and $\mathcal{P}_3(H)$, and is closed under arbitrary unions and finite intersections. Hence $(\mathcal{P}_3(H), \tau^{(3)})$ is a valid *3-superhypertopological space*.

1.3 Network Topology

A network topology is a graph structure (nodes and edges) describing connectivity, link characteristics, directions, and metrics between network devices (cf. [15–17]).

Definition 1.16 (Network Topology)**.** A *network topology* is a triple

$$G = (V, E, w),$$

where

- $V = \{v_1, v_2, \ldots, v_n\}$ is a finite set of *nodes*,
- $E \subseteq V \times V$ is a set of *edges*, each $(u, v) \in E$ indicating a direct communication link between nodes u and v,
- $w\colon E \to \mathbb{R}_{>0}$ is an optional *weight function* assigning to each edge a positive real weight (for example, capacity, latency, or cost).

If w is omitted, the network is called *unweighted*. The *adjacency matrix* $A \in \{0, 1\}^{n\times n}$ encodes the edge set by

$$A_{ij} \;=\; \begin{cases} 1, & (v_i, v_j) \in E, \\ 0, & \text{otherwise.} \end{cases}$$

Moreover, G is called *undirected* if

$$(u, v) \in E \iff (v, u) \in E,$$

and *directed* otherwise. A *path* of length k in G is a sequence $(v_{i_0}, v_{i_1}, \ldots, v_{i_k})$ with $(v_{i_{j-1}}, v_{i_j}) \in E$ for all $j = 1, \ldots, k$. The *distance* $d(u, v)$ is the minimum length of any path from u to v (if none exists, $d(u, v) = \infty$).

Example 1.17 (Small Office Network)**.** Consider a small office network with five devices:

$$V = \{\text{Router}, \text{Switch}, \text{PC}_1, \text{PC}_2, \text{Server}\}.$$

The direct links (edges) and their round-trip latencies in milliseconds are:

$$E = \{(\text{Router}, \text{Switch}), (\text{Switch}, \text{PC}_1), (\text{Switch}, \text{PC}_2), (\text{Switch}, \text{Server}), (\text{Router}, \text{Server})\},$$

with weight function w given by

$$w(\text{Router}, \text{Switch}) = 1, \quad w(\text{Switch}, \text{PC}_1) = 2, \quad w(\text{Switch}, \text{PC}_2) = 2,$$

$$w(\text{Switch}, \text{Server}) = 1, \quad w(\text{Router}, \text{Server}) = 5.$$

Since all links are bidirectional with the same latency in each direction, this is an *undirected*, *weighted* network.

The adjacency matrix $A \in \{0, 1\}^{5\times 5}$ with the node ordering $\{\text{Router}, \text{Switch}, \text{PC}_1, \text{PC}_2, \text{Server}\}$ is

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}.$$

A few illustrative paths and distances:

- A path from PC_1 to Server is
$$(\text{PC}_1, \text{Switch}, \text{Server}),$$
whose total latency is $w(\text{PC}_1, \text{Switch}) + w(\text{Switch}, \text{Server}) = 2 + 1 = 3$. Hence $d(\text{PC}_1, \text{Server}) = 3$.
- An alternative route is
$$(\text{PC}_1, \text{Switch}, \text{Router}, \text{Server}),$$
with latency $2 + 1 + 5 = 8$, which is longer and therefore not shortest.
- The distance between PC_1 and PC_2 is
$$d(\text{PC}_1, \text{PC}_2) = w(\text{PC}_1, \text{Switch}) + w(\text{Switch}, \text{PC}_2) = 2 + 2 = 4.$$

This example illustrates all aspects of Definition 1.16: a finite node set, an edge set encoding direct links, a positive weight function measuring latency, an undirected structure, an explicit adjacency matrix, and computation of paths and shortest-path distances.

Example 1.18 (Metropolitan Ring Network)**.** Consider an undirected ring network connecting six metropolitan PoPs:

$$V = \{A, B, C, D, E, F\},$$

with fiber-optic links (edges)

$$E = \{(A, B), (B, C), (C, D), (D, E), (E, F), (F, A)\}.$$

Assign to each link the one-way propagation delay (in milliseconds):

$$w(A,B) = 10, \quad w(B,C) = 20, \quad w(C,D) = 15,$$
$$w(D,E) = 10, \quad w(E,F) = 20, \quad w(F,A) = 25.$$

Since every link is bidirectional with the same delay each way, the network is *undirected* and *weighted*.

The adjacency matrix $A \in \{0,1\}^{6\times 6}$, ordered (A,B,C,D,E,F), is

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

For example, to send traffic from PoP C to PoP F, there are two possible simple paths:

$$(C,D,E,F) \quad \text{with total delay } 15 + 10 + 20 = 45 \text{ ms},$$

$$(C,B,A,F) \quad \text{with total delay } 20 + 10 + 25 = 55 \text{ ms}.$$

Hence the *distance* $d(C,F)$ is the minimum delay, namely 45 ms.

This ring topology exemplifies Definition 1.16 by specifying a finite node set, an edge set forming a cycle, a positive weight function modeling link delays, the undirected structure, the adjacency matrix, and the computation of shortest-path distances.

Theorem 1.19 (Network-Induced Topology)**.** *Let*

$$G = (V,E,w)$$

be a finite, connected, undirected network with positive edge weights $w\colon E \to \mathbb{R}_{>0}$*. Define the* shortest-path distance

$$d\colon V\times V \longrightarrow \mathbb{R}_{\geq 0}, \quad d(u,v) = \min\Big\{\sum_{i=1}^{k} w(e_i) \;\Big|\; e_1,\dots,e_k \textit{ is a path from } u \textit{ to } v\Big\}.$$

Then d is a metric on V. Let

$$\tau_d \;=\; \{U \subseteq V \mid \forall\, v\in U\ \exists r>0:\ B_d(v,r)\subseteq U\},$$

where

$$B_d(v,r) \;=\; \{u\in V \mid d(v,u) < r\}$$

is the open ball of radius r about v. Then τ_d is a topology on V, called the network topology *induced by G.*

Proof. We split the proof into two parts.

(1) d **is a metric on** V**.**

- *Nonnegativity and definiteness:* By definition each path length $\sum_i w(e_i)$ is strictly positive unless $u = v$, in which case the empty path has length 0. Hence $d(u, v) \geq 0$, with equality if and only if $u = v$.
- *Symmetry:* Since G is undirected, every path from u to v corresponds to a path of the same total weight from v to u. Thus $d(u, v) = d(v, u)$.
- *Triangle inequality:* If $P_{u\to v}$ attains the minimum length $d(u, v)$ and $P_{v\to w}$ attains $d(v, w)$, then concatenating these two paths yields a u–w path of length $d(u, v) + d(v, w)$. Hence

$$d(u, w) \;\leq\; d(u, v) + d(v, w).$$

(2) τ_d **satisfies the topology axioms.**

(a) $\varnothing$ and V are open: $\varnothing \in \tau_d$ trivially. For each $v \in V$, choose $r = 1$; then $B_d(v, 1) \subseteq V$. Hence $V \in \tau_d$.

(b) *Arbitrary unions:* Let $\{U_i\}_{i\in I} \subseteq \tau_d$. Set $U = \bigcup_{i\in I} U_i$. For any $v \in U$, there exists some index i with $v \in U_i$. Since U_i is open, choose $r > 0$ with $B_d(v, r) \subseteq U_i \subseteq U$. Thus $U \in \tau_d$.

(c) *Finite intersections:* Let $U_1, U_2 \in \tau_d$ and set $U = U_1 \cap U_2$. For $v \in U$, there exist radii $r_1, r_2 > 0$ such that

$$B_d(v, r_1) \subseteq U_1, \quad B_d(v, r_2) \subseteq U_2.$$

Taking $r = \min(r_1, r_2)$ yields $B_d(v, r) \subseteq U_1 \cap U_2 = U$. Hence $U \in \tau_d$. The same argument extends to any finite intersection.

Having verified the three defining properties of Definition 1.10, we conclude that (V, τ_d) is a topological space. □

2 Result: Network HyperTopology and Network Super-hypertopology

Network hypertopology defines a topology on subnetworks, closed under arbitrary unions and finite intersections, modeling relationships among subnetworks. Network Super-hypertopology recursively applies hypertopology to higher-order power sets of subnetworks, creating multi-level hierarchical topology spaces.

Definition 2.1 (Network Hypertopology)**.** Let $G = (V, E)$ be a network, where V is a finite set of *nodes*. Equip V with the discrete topology $\tau_V = \mathcal{P}(V)$. Denote by $\mathcal{P}(V)$ its power set (the set of all subsets of nodes). The *network hypertopology* τ_H on $\mathcal{P}(V)$ is the Vietoris topology generated by the subbasis

$$\langle U\rangle \;=\; \{\, A \subseteq V \;\mid\; A \subseteq U \},\qquad [U] \;=\; \{\, A \subseteq V \;\mid\; A \cap U \neq \varnothing \},$$

for all $U \in \tau_V$. Concretely, τ_H is the smallest topology on $\mathcal{P}(V)$ containing every $\langle U\rangle$ and $[U]$.

Example 2.2 (Network Hypertopology on a Three-Node Network)**.** Let $G = (V, E)$ be the undirected network with

$$V = \{A, B, C\}, \quad E = \{(A, B),\ (B, C)\}.$$

Equip V with the discrete topology $\tau_V = \mathcal{P}(V)$. Then the hyperspace of all node-subsets is

$$\mathcal{P}(V) = \{\varnothing, \{A\}, \{B\}, \{C\}, \{A, B\}, \{A, C\}, \{B, C\}, \{A, B, C\}\}.$$

Subset X	$X \subseteq \{A, B\}$	$X \cap \{B, C\} \neq \varnothing$
$\varnothing$	No	No
$\{A\}$	Yes	No
$\{B\}$	No	Yes
$\{C\}$	No	Yes
$\{A, B\}$	Yes	Yes
$\{A, C\}$	No	Yes
$\{B, C\}$	No	Yes
$\{A, B, C\}$	Yes	Yes

Table 1: Membership of each node subset in the Vietoris subbasis elements $\langle\{A, B\}\rangle$ and $[\{B, C\}]$.

The Vietoris subbasis on $\mathcal{P}(V)$ consists of

$$\langle U\rangle = \{\, X \subseteq V \mid X \subseteq U\}, \quad [U] = \{\, X \subseteq V \mid X \cap U \neq \varnothing\},$$

for each $U \in \tau_V$. In particular, take $U_1 = \{A, B\}$ and $U_2 = \{B, C\}$. The membership of each subset $X \subseteq V$ in the two subbasic opens can be displayed as follows:

From this table we read off:

$$\langle\{A, B\}\rangle = \{\varnothing, \{A\}, \{A, B\}, \{A, B, C\}\}, \quad [\{B, C\}] = \{\{B\}, \{C\}, \{A, B\}, \{A, C\}, \{B, C\}, \{A, B, C\}\}.$$

Their intersection,

$$\langle\{A, B\}\rangle \cap [\{B, C\}] = \{\{A, B\}, \{A, B, C\}\},$$

is a basic open in the hypertopology τ_H. Arbitrary unions and further finite intersections of such basic opens generate the full network hypertopology on $\mathcal{P}(V)$.

Example 2.3 (Network Hypertopology on a Four-Node Star Network)**.** Let $G = (V, E)$ be the undirected star network with

$$V = \{1, 2, 3, 4\}, \qquad E = \{(1, 2), (1, 3), (1, 4)\}.$$

Equip V with the discrete topology $\tau_V = \mathcal{P}(V)$. Then the hyperspace of all node-subsets is

$$\mathcal{P}(V) = \{\, X \mid X \subseteq \{1, 2, 3, 4\}\},$$

which has 16 elements.

By Definition 2.1, the Vietoris subbasis on $\mathcal{P}(V)$ consists of

$$\langle U\rangle = \{\, X \subseteq V \mid X \subseteq U\}, \quad [U] = \{\, X \subseteq V \mid X \cap U \neq \varnothing\},$$

for each $U \subseteq V$. Take for instance $U_1 = \{1, 2, 4\}$ and $U_2 = \{1, 3\}$. The following table shows membership of each subset $X \subseteq V$ in the two subbasis opens:

From Table 2 we read:

$$\langle\{1, 2, 4\}\rangle = \{\varnothing, \{1\}, \{2\}, \{4\}, \{1, 2\}, \{1, 4\}, \{2, 4\}, \{1, 2, 4\}\},$$

$$[\{1, 3\}] = \{\{1\}, \{3\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{3, 4\}, \{1, 2, 3\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\}.$$

Their intersection,

$$\langle\{1, 2, 4\}\rangle \cap [\{1, 3\}] = \{\{1\}, \{1, 2\}, \{1, 4\}, \{1, 2, 4\}\},$$

is a basic open in the network hypertopology τ_H. Arbitrary unions and further finite intersections of such basic opens generate τ_H, confirming $(\mathcal{P}(V), \tau_H)$ is a network hypertopological space.

Subset X	$X \subseteq \{1,2,4\}$	$X \cap \{1,3\} \neq \emptyset$
$\varnothing$	No	No
$\{1\}$	Yes	Yes
$\{2\}$	Yes	No
$\{3\}$	No	Yes
$\{4\}$	Yes	No
$\{1,2\}$	Yes	Yes
$\{1,3\}$	No	Yes
$\{1,4\}$	Yes	Yes
$\{2,3\}$	No	Yes
$\{2,4\}$	Yes	No
$\{3,4\}$	No	Yes
$\{1,2,3\}$	No	Yes
$\{1,2,4\}$	Yes	Yes
$\{1,3,4\}$	No	Yes
$\{2,3,4\}$	No	Yes
$\{1,2,3,4\}$	Yes	Yes

Table 2: Membership in the Vietoris subbasis sets $\langle\{1,2,4\}\rangle$ and $[\{1,3\}]$.

Theorem 2.4 (Hypertopology Axioms)**.** *The collection $\tau_H \subseteq \mathcal{P}(\mathcal{P}(V))$ defined above satisfies:*

1. $\varnothing,\ \mathcal{P}(V) \in \tau_H$.
2. *Closed under arbitrary unions.*
3. *Closed under finite intersections.*

Hence $(\mathcal{P}(V), \tau_H)$ *is a* hypertopological space.

Proof. By construction τ_H is a topology (the Vietoris topology) on $\mathcal{P}(V)$. In particular:

- $\langle\varnothing\rangle = \{\varnothing\}$ and $[\varnothing] = \varnothing$ belong to τ_H, while $\langle V\rangle = \mathcal{P}(V)$ and $[V] = \mathcal{P}(V)$, so $\varnothing, \mathcal{P}(V) \in \tau_H$.
- Unions of Vietoris-basic opens remain open in the Vietoris topology.
- Finite intersections of subbasic opens $\langle U_i\rangle$ or $[U_i]$ again have the same form:

$$\bigcap_{i=1}^{m}\langle U_i\rangle = \langle \bigcap_{i=1}^{m} U_i\rangle, \quad \bigcap_{i=1}^{m}[U_i] = [\bigcap_{i=1}^{m} U_i],$$

  and mixed intersections similarly yield unions and intersections of the U_i, all of which lie in τ_V. Thus the intersection is again in τ_H.

This verifies the three axioms. □

Theorem 2.5 (Generalization of Network Topology)**.** *The map*

$$\iota\colon V \longrightarrow \mathcal{P}(V), \qquad v \mapsto \{v\},$$

is a topological embedding of the discrete space (V, τ_V) into the hypertopological space $(\mathcal{P}(V), \tau_H)$. Consequently, the classical network node-set topology is recovered as the subspace topology on $\iota(V)$, and the network hypertopology strictly extends it.

Proof. Let $U \subseteq V$ be any open set in the discrete topology τ_V. Then

$$\iota^{-1}(\langle U\rangle) = \{\, v \in V \mid \{v\} \subseteq U \,\} = U, \quad \iota^{-1}([U]) = \{\, v \in V \mid \{v\} \cap U \neq \varnothing\} = U,$$

showing that the preimage of every Vietoris-basic open is open in τ_V. Hence ι is continuous. It is clearly injective, and the inverse $\iota(V) \to V$ is continuous by the same argument. Therefore ι is a homeomorphism onto its image $\iota(V)$. Thus the discrete topology on V embeds as the singleton-subspace of $\mathcal{P}(V)$, and τ_H genuinely generalizes the original network topology. □

Definition 2.6 (Network Super-hypertopology)**.** Let $G = (V, E)$ be a network with finite node set V. Define recursively

$$\mathcal{P}_0(V) = V, \qquad \mathcal{P}_{k+1}(V) = \mathcal{P}(\mathcal{P}_k(V)) \quad (k \geq 0).$$

Equip $\mathcal{P}_0(V)$ with the discrete topology $\tau^{(0)} = \mathcal{P}(V)$. For each $n \geq 1$, let $\tau^{(n)}$ be the Vietoris topology on $\mathcal{P}_n(V)$ generated by the subbasis of sets

$$\langle U\rangle = \{\, A \subseteq \mathcal{P}_{n-1}(V) \mid A \subseteq U\}, \quad [U] = \{\, A \subseteq \mathcal{P}_{n-1}(V) \mid A \cap U \neq \varnothing\},$$

for every open $U \in \tau^{(n-1)}$. The family $\{\,(\mathcal{P}_n(V), \tau^{(n)})\}_{n\geq 0}$ is called the *network Super-hypertopology*.

Example 2.7 (Network Super-hypertopology on a Two-Node Network)**.** Let $G = (V, E)$ be the simple undirected network with

$$V = \{A, B\}, \qquad E = \{(A, B)\}.$$

We build the sequence $\{(\mathcal{P}_n(V), \tau^{(n)})\}_{n\geq 0}$ of superhypertopological spaces.

Level 0. The base set and its discrete topology:

$$\mathcal{P}_0(V) = V = \{A, B\}, \qquad \tau^{(0)} = \mathcal{P}(\mathcal{P}_0(V)) = \{\varnothing, \{A\}, \{B\}, \{A, B\}\}.$$

Level 1.

$$\mathcal{P}_1(V) = \mathcal{P}(V) = \{\varnothing, \{A\}, \{B\}, \{A, B\}\}.$$

The Vietoris subbasis on $\mathcal{P}_1(V)$ is

$$\langle U\rangle = \{X \subseteq V \mid X \subseteq U\}, \quad [U] = \{X \subseteq V \mid X \cap U \neq \varnothing\}, \quad U \in \tau^{(0)}.$$

For $U = \{A\}$ and $U' = \{B\}$, membership is:

Thus $\langle\{A\}\rangle = \{\varnothing, \{A\}\}$ and $[\{B\}] = \{\{B\}, \{A, B\}\}$. A basic open is, for example,

$$\langle\{A\}\rangle \cap [\{B\}] = \{\{A, B\}\}.$$

Subset $X \subseteq V$	$X \subseteq \{A\}$	$X \cap \{B\} \neq \emptyset$
$\varnothing$	No	No
$\{A\}$	Yes	No
$\{B\}$	No	Yes
$\{A, B\}$	No	Yes

Table 3: Vietoris subbasis on $\mathcal{P}_1(V)$ for $U = \{A\}, U' = \{B\}$.

$\mathcal{X} \subseteq \mathcal{P}_1(V)$	$\mathcal{X} \subseteq \{\{A\}\}$	$\mathcal{X} \cap \{\{A\}\} \neq \emptyset$
$\varnothing$	Yes	No
$\{\varnothing\}$	No	No
$\{\{A\}\}$	Yes	Yes
$\{\{B\}\}$	No	No
$\{\{A, B\}\}$	No	No
$\{\varnothing, \{A\}\}$	No	Yes
$\{\{A\}, \{B\}\}$	No	Yes
$\{\{A\}, \{A, B\}\}$	No	Yes
$\{\varnothing, \{A\}, \{A, B\}\}$	No	Yes
$\mathcal{P}_1(V)$	No	Yes

Table 4: Vietoris subbasis on $\mathcal{P}_2(V)$ for $\mathcal{U} = \{\{A\}\}$.

Level 2.

$$\mathcal{P}_2(V) = \mathcal{P}(\mathcal{P}_1(V)),$$

consisting of all 16 subsets of $\{\varnothing, \{A\}, \{B\}, \{A, B\}\}$. Its Vietoris subbasis is

$$\langle \mathcal{U} \rangle = \{\mathcal{X} \subseteq \mathcal{P}_1(V) \mid \mathcal{X} \subseteq \mathcal{U}\}, \quad [\mathcal{U}] = \{\mathcal{X} \subseteq \mathcal{P}_1(V) \mid \mathcal{X} \cap \mathcal{U} \neq \varnothing\},$$

for each $\mathcal{U} \in \tau^{(1)}$. Take $\mathcal{U} = \{\{A\}\}$. Then:

Hence $\langle \{\{A\}\} \rangle = \{\varnothing, \{\{A\}\}\}$ and $[\{\{A\}\}] = \{\{\{A\}\}, \{\varnothing, \{A\}\}, \{\{A\}, \{A, B\}\}, \ldots\}$. A basic open example is

$$\langle \{\{A\}\} \rangle \cap [\{\{A, B\}\}] = \{\{\{A\}, \{A, B\}\}\}.$$

Each $\tau^{(n)}$ contains $\varnothing$ and $\mathcal{P}_n(V)$, and is closed under arbitrary unions and finite intersections, so $\{(\mathcal{P}_n(V), \tau^{(n)})\}$ defines a valid network Super-hypertopology.

Example 2.8 (Network Super-hypertopology on a Triangle Network)**.** Let $G = (V, E)$ be the undirected triangle network with

$$V = \{A, B, C\}, \quad E = \{(A, B), (B, C), (C, A)\}.$$

We construct the Super-hypertopology $\{(\mathcal{P}_n(V), \tau^{(n)})\}_{n \geq 0}$.

Level 0.

$$\mathcal{P}_0(V) = V, \quad \tau^{(0)} = \mathcal{P}(V) = \{\varnothing, \{A\}, \{B\}, \{C\}, \{A, B\}, \{B, C\}, \{C, A\}, \{A, B, C\}\}.$$

Level 1.

$$\mathcal{P}_1(V) = \mathcal{P}(\mathcal{P}_0(V)),$$

the 8 subsets of V. The Vietoris subbasis on $\mathcal{P}_1(V)$ is

$$\langle U \rangle = \{X \subseteq V \mid X \subseteq U\}, \quad [U] = \{X \subseteq V \mid X \cap U \neq \emptyset\}, \quad U \in \tau^{(0)}.$$

Take $U_1 = \{A, B\}$ and $U_2 = \{B, C\}$. Table 5 lists membership of each $X \subseteq V$ in these subbasis opens.

Subset X	$X \subseteq \{A, B\}$	$X \cap \{B, C\} \neq \emptyset$
$\emptyset$	No	No
$\{A\}$	Yes	No
$\{B\}$	Yes	Yes
$\{C\}$	No	Yes
$\{A, B\}$	Yes	Yes
$\{B, C\}$	No	Yes
$\{C, A\}$	No	No
$\{A, B, C\}$	No	Yes

Table 5: Level 1 Vietoris subbasis for $U_1 = \{A, B\}$ and $U_2 = \{B, C\}$.

Hence

$$\langle\{A, B\}\rangle = \{\emptyset, \{A\}, \{B\}, \{A, B\}\}, \quad [\{B, C\}] = \{\{B\}, \{C\}, \{A, B\}, \{B, C\}, \{A, B, C\}\}.$$

A basic open is

$$\langle\{A, B\}\rangle \cap [\{B, C\}] = \{\{B\}, \{A, B\}\}.$$

Level 2.

$$\mathcal{P}_2(V) = \mathcal{P}(\mathcal{P}_1(V)),$$

with $2^8 = 256$ elements. Its Vietoris subbasis is

$$\langle \mathcal{U} \rangle = \{\mathcal{X} \subseteq \mathcal{P}_1(V) \mid \mathcal{X} \subseteq \mathcal{U}\}, \quad [\mathcal{U}] = \{\mathcal{X} \subseteq \mathcal{P}_1(V) \mid \mathcal{X} \cap \mathcal{U} \neq \emptyset\},$$

for each $\mathcal{U} \in \tau^{(1)}$. Let $\mathcal{U} = \{\{B\}, \{A, B\}\}$. Table 6 displays membership for a selection of $\mathcal{X} \subseteq \mathcal{P}_1(V)$.

$\mathcal{X}$	$\mathcal{X} \subseteq \mathcal{U}$	$\mathcal{X} \cap \mathcal{U} \neq \emptyset$
$\emptyset$	Yes	No
$\{\{B\}\}$	Yes	Yes
$\{\{A\}\}$	No	No
$\{\{A, B\}\}$	Yes	Yes
$\{\{C\}\}$	No	No
$\{\{B\}, \{C\}\}$	No	Yes
$\{\{A, B\}, \{C\}\}$	No	Yes
$\{\{B\}, \{A, B\}\}$	Yes	Yes
$\mathcal{P}_1(V)$	No	Yes

Table 6: Level 2 Vietoris subbasis for $\mathcal{U} = \{\{B\}, \{A, B\}\}$.

Thus

$$\langle \mathcal{U} \rangle = \{\varnothing, \{\{B\}\}, \{\{A, B\}\}, \{\{B\}, \{A, B\}\}\}, \quad [\mathcal{U}] = \{\mathcal{X} \mid \mathcal{X} \cap \mathcal{U} \neq \emptyset\}.$$

A basic open is, for example,

$$\langle \mathcal{U} \rangle \cap [\{\{A\}, \{C\}\}],$$

which selects those $\mathcal{X} \subseteq \mathcal{P}_1(V)$ lying in $\mathcal{U}$ and meeting $\{\{A\}, \{C\}\}$.

Each $\tau^{(n)}$ contains $\varnothing$ and $\mathcal{P}_n(V)$ and is closed under arbitrary unions and finite intersections. Therefore $\{(\mathcal{P}_n(V), \tau^{(n)})\}_{n \geq 0}$ forms a valid network superhypertopological space.

Theorem 2.9 (Generalization of Network Hypertopology)**.** *For each $n \geq 0$, the singleton-inclusion map*

$$\eta_n : \mathcal{P}_n(V) \longrightarrow \mathcal{P}_{n+1}(V), \qquad A \mapsto \{A\},$$

is a topological embedding of $(\mathcal{P}_n(V), \tau^{(n)})$ into $(\mathcal{P}_{n+1}(V), \tau^{(n+1)})$. In particular, the network hypertopology $\tau^{(1)}$ on $\mathcal{P}_1(V)$ appears as the subspace topology on $\eta_0(V)$, and more generally $\tau^{(n)}$ embeds into $\tau^{(n+1)}$. Hence the Super-hypertopology strictly extends the hypertopology at every level.

Proof. Let $U \subseteq \mathcal{P}_n(V)$ be open in $\tau^{(n)}$. Then by definition of the Vietoris subbasis,

$$\eta_n^{-1}(\langle U \rangle) = \{ A \mid \{A\} \subseteq U \} = U, \quad \eta_n^{-1}([U]) = \{ A \mid \{A\} \cap U \neq \varnothing \} = U,$$

showing that η_n is continuous. It is injective, and its inverse on $\eta_n(\mathcal{P}_n(V))$ is continuous by the same reasoning. Thus η_n is a homeomorphism onto its image. □

Theorem 2.10 (Super-hypertopology Axioms)**.** *For each $n \geq 1$, the Vietoris topology $\tau^{(n)}$ on $\mathcal{P}_n(V)$ satisfies:*

1. *$\varnothing$, $\mathcal{P}_n(V) \in \tau^{(n)}$.*
2. *Closed under arbitrary unions.*
3. *Closed under finite intersections.*

Consequently, $\{ (\mathcal{P}_n(V), \tau^{(n)})\}_{n \geq 1}$ forms a valid superhypertopological space.

Proof. We argue by induction on n.

Base case ($n = 1$): $\tau^{(1)}$ is the standard Vietoris (network hypertopology) on $\mathcal{P}_1(V) = \mathcal{P}(V)$. It is well-known to contain $\varnothing$ and $\mathcal{P}(V)$, be closed under arbitrary unions of Vietoris-basic opens, and closed under finite intersections of subbasic opens $\langle U \rangle$ and $[U]$.

Inductive step: Assume $\tau^{(n)}$ on $\mathcal{P}_n(V)$ satisfies the three axioms. By definition, $\tau^{(n+1)}$ is generated by the subbasis $\{\langle U \rangle, [U] \mid U \in \tau^{(n)}\}$. Any union of such subbasic opens is open by construction. A finite intersection of subbasis elements has the form

$$\langle U_1 \rangle \cap \cdots \cap \langle U_m \rangle = \langle U_1 \cap \cdots \cap U_m \rangle, \quad [U_1] \cap \cdots \cap [U_m] = [U_1 \cap \cdots \cap U_m],$$

and mixed intersections likewise reduce to unions and intersections of finitely many $U_i \in \tau^{(n)}$. Since $\tau^{(n)}$ is closed under these operations, the resulting set lies in the subbasis, hence in $\tau^{(n+1)}$. Finally, $\langle \varnothing \rangle = \{\varnothing\}$ and $[\varnothing] = \varnothing$ are in $\tau^{(n+1)}$, and $\langle \mathcal{P}_n(V) \rangle = [\mathcal{P}_n(V)] = \mathcal{P}_{n+1}(V)$. Thus $\tau^{(n+1)}$ satisfies all axioms, completing the induction. □

3 Conclusion and Future Work

In this work, we have presented the concepts of *Network Hypertopology* and *Network Super-hypertopology*, which extend the traditional graph-based network topology into hierarchically structured topologies on node power sets.

For future research, we intend to integrate uncertainty-handling frameworks such as Fuzzy Sets [18], Intuitionistic Fuzzy Sets [19], Hesitant Fuzzy Sets [20], Picture Fuzzy Sets [21], HyperFuzzy Sets [22], Neutrosophic Sets [23, 24], Double-valued Neutrosophic Sets [25], and Plithogenic Sets [26] to model multi-valued relationships under uncertainty. We also plan to conduct computational experiments to demonstrate practical applications and to validate the theoretical frameworks introduced here. Additionally, we envision further extensions employing HyperGraphs [27] and SuperHyperGraphs [28] to deepen and broaden the scope of our approach.

Funding

This work was carried out without external financial support.

Acknowledgments

We are grateful to all colleagues and reviewers whose feedback and expertise improved this manuscript. We also thank the authors of the cited literature for establishing the foundations upon which this study builds, and our institutions for providing the resources that enabled this research.

Data Availability

No data were generated or analyzed in the course of this theoretical investigation. We encourage future empirical studies to test and extend the ideas presented here.

Ethical Approval

Not applicable.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Disclaimer

The concepts introduced in this paper are theoretical and have not yet been validated empirically. Readers should independently verify all references and may encounter inadvertent errors. The views expressed here are solely those of the authors and do not necessarily reflect the positions of their affiliated organizations.

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