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Analysis of Wave Transformational Method to Nonlinear PDEs: Exponential Rational Function Method

Kirti[†]

[†]Department of Mathematics, Central University of Haryana,

Jant-Pali, Mahendergarh-123029, Haryana, India.

Corresponding Author: bhardwajkirti718@gmail.com

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Abstract

This work explores the well-known wave transformational approach as exponential rational function method (ERFM) to the nonlinear partial differential equations (PDEs). The method utilizes a rational function in term of the exponential functions, which is suitable for the study of nonlinear equation. Due to the rational function solutions, the technique can give different type of solutions such as solitons, lumps, kinks, and breathers. As it considers the exponential rational forms, it provides the exact solution of the nonlinear PDEs. This study investigates the ERFM to the well-known equations such as KdV equation, KP equation, mKdV equation, and KMN equation. It also analyses the dynamical behavior for the obtained solutions in three dimensional graphics for appropriate values of the arbitrary parameters. The models studied in this work explains the nonlinear wave phenomena from different fields such as fluid dynamics, oceanography, plasma physics, opitic fibers, and other sciences.

1 Introduction

In mathematics and physics, a nonlinear equation [1–7] play major role to solve complex wave phenomena in nonlinear sciences such as plasma physics, optical fibers, solid state physics chemical physics. Its help to understand stability and wave interactions. The exponential function used to find the exact solution of nonlinear evolution equations. The method reduces the partial differential equations (PDEs) to an ordinary differential equations (ODEs) by using a traveling wave transformation. Nonlinear evaluation equations are important in theoretical and practical applications. This method is very useful in mathematical physics and applied mathematics [1], and was developed for solving PDEs utilizing wave transform.ation. It assumes the solution of the exponential rational function which enable to drive soliton, periodic and singular wave solutions.

Uses:

1. It finds exact solutions of nonlinear evolution equations such as korteweg-de vries(KdV) [8,10], Modified korteweg-de vrie (mkdv) equation [18], Kundu-Mukerjee Naskar (KMN) equation [9], Kadomtsev-Petviashvili (KP) equation [15, 21], and others.
2. These solution play important role of physical phenomena like as solitary wave, plasma wave, optical pulse propagation.
3. It is more systematic and unified as compare to other methods. It can generate various type of wave solutions such as solitons, lumps, kinks, and breathers.

Several researchers have developed various method to obtains exact solution of nonlinear evolutions equations such as ,Hirota bilinear method [4], The tanh-sech method [5, 6], homogeneous balance method [7, 8], the (G/G)-expansion method, inverse scattering method [2], extended tanh method [6], sine–cosine method [20, 21], pseudo-spectral method [12], Lie group analysis [13], and others techniques.

2 General description of ERFM

We consider the wave transformation

$$v(x_1, x_2, x_3, \cdots, t) = V(\phi); \qquad \phi = a_1x_1 + a_2x_2 + \cdots + a_nx_n + bt, \tag{1}$$

to the $(n + 1)$-dimensional nonlinear partial differential equation

$$D(v, v_t, v_{x_1}, v_{tt}, v_{x_1x_2}, v_{x_2x_2}, \ldots) = 0, \tag{2}$$

where D is a polynomial function of dependent and independent variables and their partial derivatives, to covert PDE to an ODE as

$$H(V, V', V'', V''', \ldots) = 0. \tag{3}$$

We consider the general solution of equation (3) as

$$\pi(\phi) = \frac{c_0 + c_1\pi(\phi) + c_2\pi'(\phi) + \ldots + c_n\pi^n(\phi)}{d_0 + d_1\pi(\phi) + d_2\pi'(\phi) + \ldots + d_n\pi^n(\phi)}, \tag{4}$$

where

$$\pi(\phi) = \frac{x_1e^{b_1(\phi)} + x_2e^{b_2(\phi)}}{x_3e^{b_3(\phi)} + x_4e^{b_4(\phi)}}.$$

having c_i ,d_i as the unknown coefficients for $i = 0, 1, \ldots, N$ and x_i, b_i are arbitrary constants(i=1,2,3,4). N is a positive integer computed using homogeneous balancing principle in the equation (1).
We put the equation (4) into equation (3), and collect the cofficients to exponential term to get the values of parameters by solving the obtained algebraic system.

3 Application of ERFM to nonlinear PDEs

3.1 Korteweg-de Vries (KdV) equation

Let us consider the KdV equation [14]

$$v_t + 6vv_x + v_{xxx} = 0. \tag{5}$$

We consider the wave transformation

$$v(x,t) = V(\phi), \;\; \phi = x - ct, \tag{6}$$

where ϕ is a traveling wave variable, c is wave speed, and $V(\phi)$ is unknown function.
Now, we get

$$v_t(x,t) = -cV'(\phi),$$
$$v_x(x,t) = V'(\phi),$$
$$v_{xx}(x,t) = V''(\phi),$$
$$v_{xxx}(x,t) = V'''(\phi).$$

On putting these values in equation (5), we get

$$-cV'(\phi) + 6V(\phi)V'(\phi) + V'''(\phi) = 0. \tag{7}$$

Applying ERFM and consider

$$V(\phi) = A + \frac{Be^{\xi}}{(1+e^{\xi})^2}, \;\; \xi = \lambda\phi. \tag{8}$$

and get

$$V'(\phi) = \frac{B\lambda e^{\xi}(1-e^{\xi})}{(1+e^{\xi})^3},$$
$$V''(\phi) = \frac{B\lambda^2 e^{\xi}\left((e^{\xi})^2 - 4e^{\xi} + 1\right)}{(1+e^{\xi})^4},$$
$$V'''(\phi) = \frac{B\lambda^3 e^{\xi}\left(-(e^{\xi})^3 + 11(e^{\xi})^2 - 11e^{\xi} + 1\right)}{(1+e^{\xi})^5}.$$

On substituting the value of V', V'', V''' in equation (7), and compute the value of A,B, c and λ,we get

$$-c\left(\frac{B\lambda e^{\xi}(1-e^{\xi})}{(1+e^{\xi})^3}\right) + 6\left(\frac{B\lambda e^{\xi}(1-e^{\xi})}{(1+e^{\xi})^3}\right)\left(A + \frac{Be^{\xi}}{(1+e^{\xi})^2}\right) + \frac{B\lambda^3 e^{\xi}\left(-(e^{\xi})^3 + 11(e^{\xi})^2 - 11e^{\xi} + 1\right)}{(1+e^{\xi})^5} = 0, \tag{9}$$

$$(e^{\lambda\phi} - 1)[(6A - c + \lambda^2)((e^{\xi})^2 + 1) + (12A + 6B - 2c + 10\lambda^2)e^{\xi}] = 0. \tag{10}$$

Comparing the coefficient of $(e^{\xi})^2$ and e^{ξ} to zero, for computing the value of A,B,c and λ as

$$6A - c + \lambda^2 = 0, \quad 12A + 6B - 2c - 10\lambda^2 = 0. \tag{11}$$

For $A = 0$,, we obtain $B = 2c, \;\; c = \lambda^2$.
Putting the values of A, B, c, and λ in equation (8), we get

$$V(\phi) = \frac{2ce^{\lambda\phi}}{(1+e^{\lambda\phi})^2} \tag{12}$$

$$V(\phi) = \frac{c}{2}\operatorname{sech}^2\left(\frac{\sqrt{c}\phi}{2}\right). \tag{13}$$

Put the value ϕ in the above equation,we get

$$v(x,t) = \frac{c}{2}\operatorname{sech}^2\left(\frac{\sqrt{c}}{2}(x-ct)\right), \tag{14}$$

which gives the required solution of the equation (5), and having dynamics in Figure 1.

3.2 Modified Korteweg-de Vries (mKdV) equation

Let us consider the mKdV equation [18]

$$u_t + 6u^2u_x + u_{xxx} = 0. \tag{15}$$

Using wave transformation

$$u(x,t) = U(\phi), \;\; \phi = x - ct, \tag{16}$$

where ϕ is traveling wave variable, c is wave speed, and $U(\phi)$ is an unknown function.
We have

$$u_t(x,t) = -cU'(\phi),$$
$$u_x(x,t) = U'(\phi),$$
$$u_{xxx}(x,t) = U'''.$$

Putting all these values in the equation (15),we get

$$-cU' + 6U^2U' + U''' = 0. \tag{17}$$

Applying ERFM and consider

$$U(\phi) = A + \frac{B\sqrt{e^{\xi}}}{1+e^{\xi}}, \;\; \xi = \lambda\phi. \tag{18}$$

or

$$U(\phi) = A + \frac{BS}{1+S^2}, \quad S = e^{\frac{\lambda\phi}{2}}. \tag{19}$$

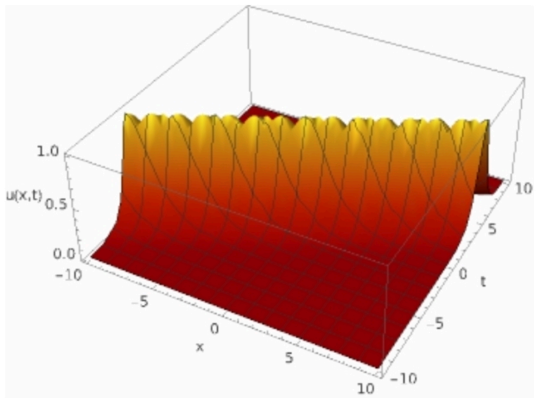
Now, we have

$$U'(\phi) = \frac{B\lambda}{2}\frac{S(1-S^2)}{(1+S^2)^2},$$
$$U''(\phi) = \frac{B\lambda^2}{4}\frac{S(S^4-6S^2+1)}{(1+S^2)^3}.$$
$$U'''(\phi) = \frac{B\lambda^3}{8}\frac{S(1-23S^2+23S^4-S^6)}{(1+S^2)^4}$$

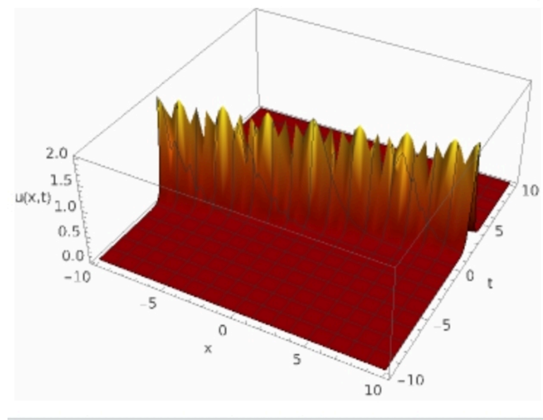
Substituting the value of U', U'', U''' in equation (17) and compute the value of A,B,c and λ, we get

$$-c\left(\frac{B\lambda}{2}\frac{S(1-S^2)}{(1+S^2)^2}\right) + 6\left(A + \frac{BS}{1+S^2}\right)^2\left(\frac{B\lambda}{2}\frac{S(1-S^2)}{(1+S^2)^2}\right) + \frac{B\lambda^3}{8}\frac{S(1-23S^2+23S^4-S^6)}{(1+S^2)^4} = 0. \tag{20}$$

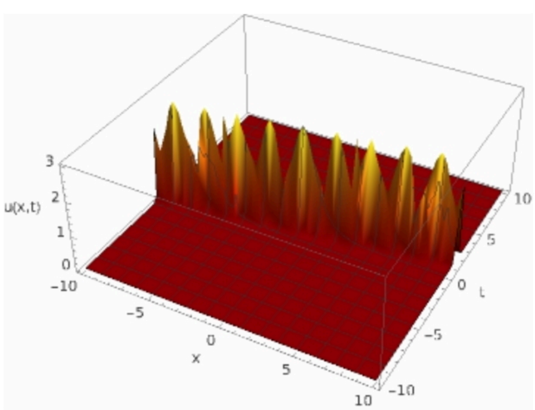
$$4A(2A^2-c) + B(24A^2-4c+\lambda^2)S + 12A(2A^2+2B^2-c)S^2 + 2B(24A^2+4B^2-4c-3\lambda^2)S^3$$



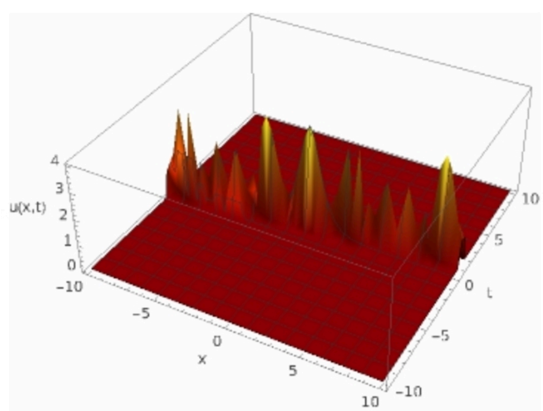
(a) c = 2



(b) c = 4



(c) c = 6



(d) c = 8

Figure 1: Nonlinear wave profiles of solution (14) for different values of c.

$$+12A(2A^2+2B^2-c)S^4+B(24A^2-4c+\lambda^2)S^5+4A(2A^2-c)S^6=0. \tag{21}$$

Comparing the coefficients of exponential functions to zero,we get

$$4A(2A^2-c)=0,$$

$$B(24A^2-4c+\lambda^2)=0,$$

$$12A(2A^2+2B^2-c)=0,$$

$$2B(24A^2+4B^2-4c-3\lambda^2)=0. \tag{22}$$

For $A=0$, we obtain $B^2=4c$,$\lambda^2=4c$.
On putting the values of $A,B,c,$ and λ in equation (18), we get

$$U(\phi)=B\left(\frac{e^{\frac{\lambda\phi}{2}}}{1+e^{\lambda\phi}}\right)=B\left(\frac{1}{2}\operatorname{sech}\frac{\lambda\phi}{2}\right)=\sqrt{c}\operatorname{sech}\left(\sqrt{c}(\phi)\right) \tag{23}$$

or

$$u(x,t)=\sqrt{c}\operatorname{sech}\left(\sqrt{c}(x-ct)\right), \tag{24}$$

which is the desired solution of equation (15), and having dynamics in Figure 2.

3.3 Kundu-Mukerjee-Naskar (KMN) equation

The KMN [9] equation is generalized form of KdV equation

$$u_t+uu_x+u_{xxx}+u_{xy}=0. \tag{25}$$

With wave transformation,

$$u(x,y,t)=U(\phi), \quad \phi=x+y-ct, \tag{26}$$

we have

$$u_x(x,y,t)=c\,U'(\phi),$$

$$u_t(x,y,t)=U'(\phi),$$

$$u_{xy}(x,y,t)=U''(\phi),$$

$$u_{xxx}(x,y,t)=U'''(\phi).$$

Putting all these values in equation (25), we get

$$-cU'(\phi)+U(\phi)U'(\phi)+U'''(\phi)+U''(\phi)=0. \tag{27}$$

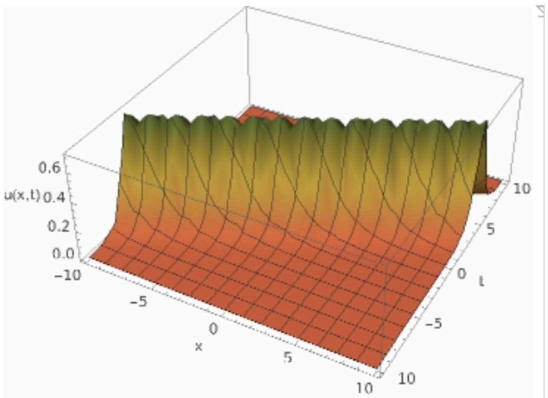
Applying ERFM and consider

$$U(\phi)=\frac{a_0+a_1e^{\xi}}{1+b_1e^{\xi}}, \quad \xi=\lambda\phi \tag{28}$$

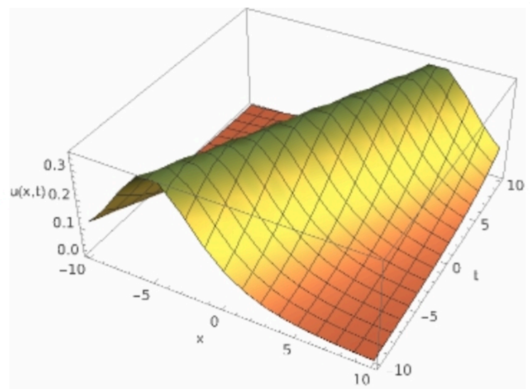
we have

$$U'(\phi)=\frac{a_1\lambda e^{\lambda\phi}(1+b_1e^{\lambda\phi})-(a_0+a_1e^{\lambda\phi})(b_1\lambda e^{\lambda\phi})}{(1+b_1e^{\lambda\phi})^2}$$

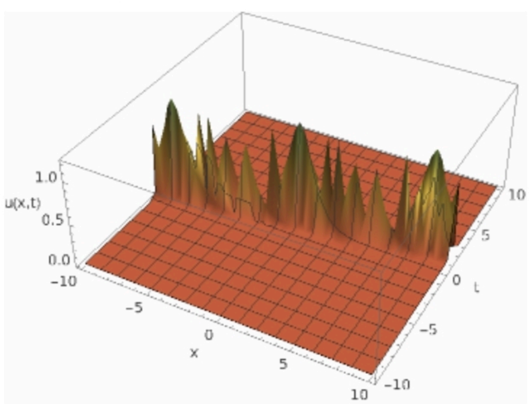
$$U'(\phi)=\frac{\lambda e^{\lambda\phi}(a_1-a_0b_1)}{(1+b_1e^{\lambda\phi})^2}.$$



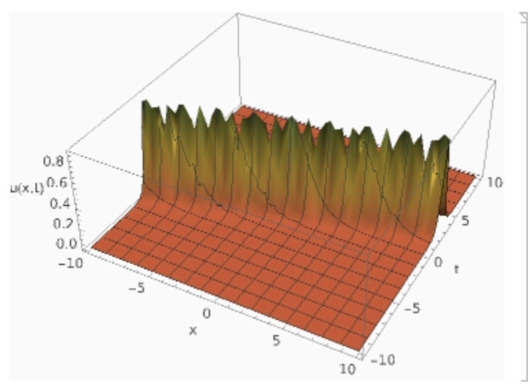
(a) c = 2



(b) c = 0.5



(c) c = 6



(d) c = 3.4

Figure 2: Nonlinear wave profiles of solution (24) for different values of c

$$U''(\phi) = \frac{\lambda^2 e^{\lambda\phi}(a_1 - a_0 b_1)(1 - b_1 e^{\lambda\phi})}{(1 + b_1 e^{\lambda\phi})^3}.$$

$$U'''(\phi) = \frac{\lambda^3\left(a_1 - a_0 b_1\right) e^{\lambda\phi}\left(1 - 4b_1 e^{\lambda\phi} + b_1^2 e^{2\lambda\phi}\right)}{\left(1 + b_1 e^{\lambda\phi}\right)^4}.$$

Substituting the values U', U'', U''' in equation (27) and compute the value of a_0, a_1, b_1, we get:

$$-c\left(\frac{a_1\lambda e^{\lambda\phi}(1 + b_1 e^{\lambda\phi}) - (a_0 + a_1 e^{\lambda\phi})(b_1\lambda e^{\lambda\phi})}{(1 + b_1 e^{\lambda\phi})^2}\right) + \left(\frac{a_1\lambda e^{\lambda\phi}(1 + b_1 e^{\lambda\phi}) - (a_0 + a_1 e^{\lambda\phi})(b_1\lambda e^{\lambda\phi})}{(1 + b_1 e^{\lambda\phi})^2}\right)$$

$$\left(\frac{a_0 + a_1 e^{\xi}}{1 + b_1 e^{\xi}}\right) + \frac{\lambda^3\left(a_1 - a_0 b_1\right) e^{\lambda\phi}\left(1 - 4b_1 e^{\lambda\phi} + b_1^2 e^{2\lambda\phi}\right)}{\left(1 + b_1 e^{\lambda\phi}\right)^4} + \frac{\lambda^2 e^{\lambda\phi}(a_1 - a_0 b_1)(1 - b_1 e^{\lambda\phi})}{(1 + b_1 e^{\lambda\phi})^3} = 0. \quad (29)$$

Assuming $a_0 = 0, a_1 = c, b_1 = 1.$, and putting these values in equation (28), we get

$$U(\phi) = \frac{ce^{\xi}}{1 + e^{\xi}} = \frac{c}{1 + e^{-\xi}} = \frac{c}{2}\operatorname{sech}^2\left(\frac{\xi}{2}\right)$$

Let $\lambda = \sqrt{c}$, with $\xi = \lambda\phi$, we get

$$U(\phi) = \frac{c}{2}\operatorname{sech}^2\left(\frac{\sqrt{c}}{2}\phi\right)$$

or

$$u(x, y, t) = \frac{c}{2}\operatorname{sech}^2\left(\frac{\sqrt{c}}{2}(x + y - ct).\right), \quad (30)$$

which gives the solution for equation (**??**), and dynamics in Figure 3.

3.4 Kadomtsev-Petviashvili (KP) equation

Let us consider KP equation [15, 19]

$$(u_t + 6uu_x + u_{xxx})_x + 3\sigma^2 u_{yy} = 0, \quad (31)$$

where $\sigma = \pm 1$. Let us considering the transformation

$$u(x, y, t) = U(\phi), \quad \phi = x + \alpha y - ct. \quad (32)$$

So, we have

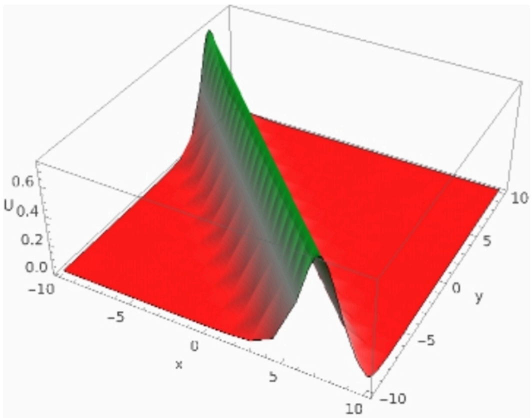
$$u_t(x, y, t) = -cU'(\phi),$$
$$u_x(x, y, t) = U'(\phi),$$
$$u_{yy}(x, y, t) = \alpha^2 U''(\phi),$$
$$u_{xxx}(x, y, t) = U'''(\phi).$$

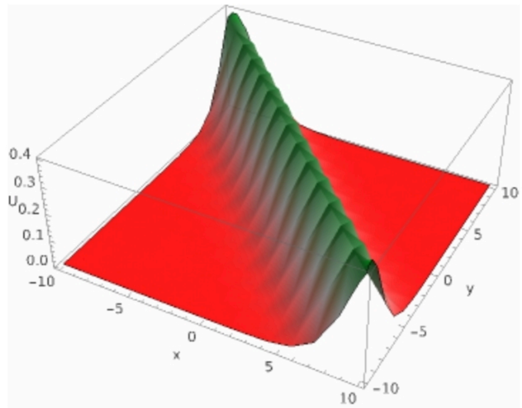
On putting these values in equation (31), we get

$$-cU''(\phi) + 6U'(\phi)^2 + 6U(\phi)U''(\phi) + U''''(\phi) + 3\sigma^2\alpha^2 U''(\phi) = 0. \quad (33)$$

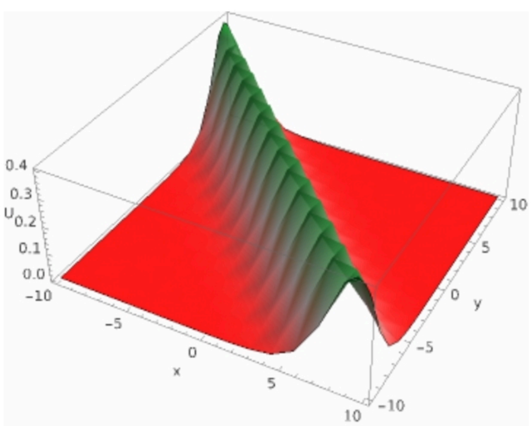
$$\Rightarrow \quad U'''' + 6UU'' + 6(U')^2 + \left(3\sigma^2\alpha^2 - c\right)U'' = 0. \quad (34)$$

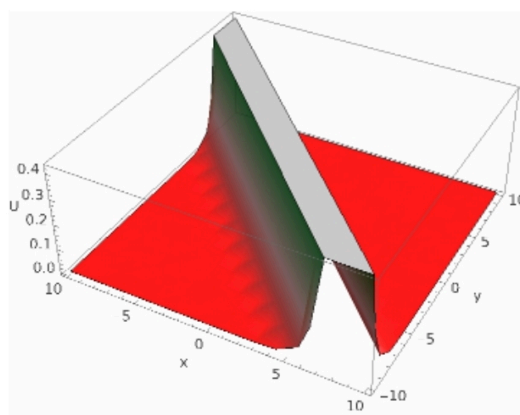
$$\Rightarrow \quad U'''' + 6UU'' + 6(U')^2 + AU'' = 0, \quad A = 3\sigma^2\alpha^2 - c. \quad (35)$$





(a) c = 1.5 (b) c = 0.8





(c) c = 0.5 (d) c = 2.5

Figure 3: Nonlinear wave profiles of solution (30) for different values of c.

For $\sigma = \alpha = 1; A = 3 - c$. Applying ERFM, and consider

$$U(\phi) = \frac{a_0 + a_1 e^{\xi}}{1 + b_1 e^{\xi}}, \;\; \xi = \lambda\phi, \tag{36}$$

we have

$$U'(\phi) = \frac{a_1 \lambda e^{\lambda\phi}\left(1 + b_1 e^{\lambda\phi}\right) - \left(a_0 + a_1 e^{\lambda\phi}\right)\left(b_1 e^{\lambda\phi}\right)}{(1 + b_1 e^{\lambda\phi})^2},$$

$$U''(\phi) = \frac{\lambda^2 e^{\lambda\phi}(a_1 - a_0 b_1)(1 - b_1 e^{\lambda\phi})}{(1 + b_1 e^{\lambda\phi})^3},$$

$$U'''(\phi) = \frac{\lambda^3\left(a_1 - a_0 b_1\right) e^{\lambda\phi}\left(1 - 4b_1 e^{\lambda\phi} + b_1^2 e^{2\lambda\phi}\right)}{\left(1 + b_1 e^{\lambda\phi}\right)^4},$$

$$U^{(4)}(\phi) = \frac{\lambda^4\left(a_1 - a_0 b_1\right) e^{\lambda\phi}\left(1 - 11b_1 e^{\lambda\phi} + 11b_1^2 e^{2\lambda\phi} - b_1^3 e^{3\lambda\phi}\right)}{\left(1 + b_1 e^{\lambda\phi}\right)^5}.$$

Putting all these values in equation (35) and compute the value of a_0, a_1, and b_1, we get

$$\frac{\lambda^4\left(a_1 - a_0 b_1\right) e^{\lambda\phi}\left(1 - 11b_1 e^{\lambda\phi} + 11b_1^2 e^{2\lambda\phi} - b_1^3 e^{3\lambda\phi}\right)}{\left(1 + b_1 e^{\lambda\phi}\right)^5} + 6\left(\frac{a_0 + a_1 e^{\xi}}{1 + b_1 e^{\xi}}\right)$$

$$\left(\frac{\lambda^2 e^{\lambda\phi}(a_1 - a_0 b_1)(1 - b_1 e^{\lambda\phi})}{(1 + b_1 e^{\lambda\phi})^3}\right) + 6\left(\frac{a_1 \lambda e^{\lambda\phi}\left(1 + b_1 e^{\lambda\phi}\right) - \left(a_0 + a_1 e^{\lambda\phi}\right)\left(b_1 e^{\lambda\phi}\right)}{\left(1 + b_1 e^{\lambda\phi}\right)^2}\right)^2$$

$$+A\left(\frac{\lambda^2 e^{\lambda\phi}(a_1 - a_0 b_1)(1 - b_1 e^{\lambda\phi})}{(1 + b_1 e^{\lambda\phi})^3}\right) = 0. \tag{37}$$

Assuming $a_0 = 0, \; a_1 = c, \; b_1 = 1$.
From equation (36), we get

$$U(\phi) = \frac{ce^{\xi}}{1 + e^{\xi}} = \frac{c}{1 + e^{-\xi}} = \frac{c}{2}\operatorname{sech}^2 \lambda\phi. \tag{38}$$

Let $\lambda = \frac{\sqrt{c}}{2}$, with $\xi = \lambda\phi$, we have

$$u(x, y, t) = \frac{c}{2} sech^2\left(\frac{\sqrt{c}}{2}\left(x + y - ct\right)\right), \tag{39}$$

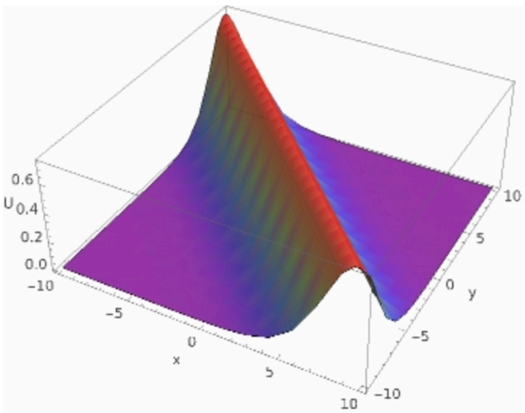
that gives the solution for equation (31), and dynamics are shown in Figure 4.

4 Advantages and limitation of ERFM

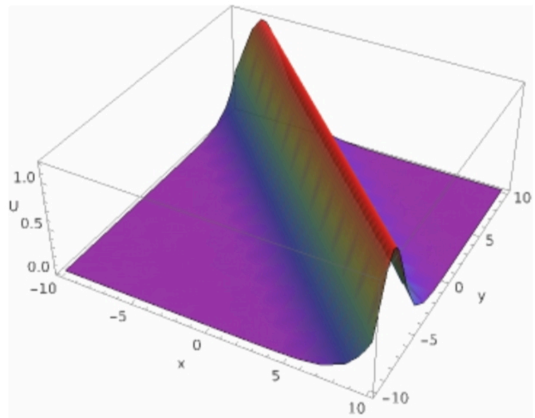
4.1 Advantages

- The method helps to get clear, step-by-step answer for exact solutions that are useful to understand the system behavior.
- It can work for several kinds of nonlinear partial differential equations.
- This approach can investigate the solutions more quickly compared to other methods.

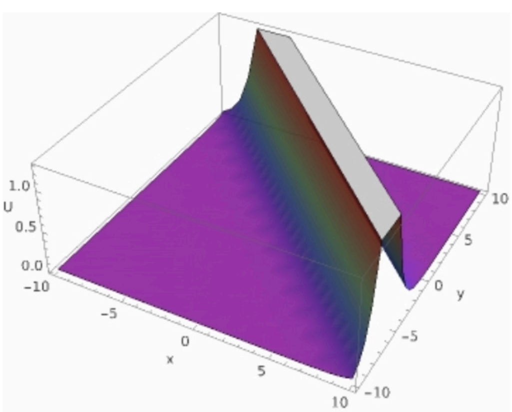
11.pdf



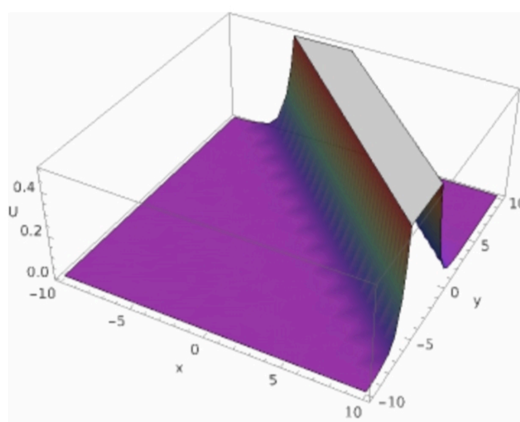
(a) c = 0.5



(b) c = 0.8



(c) c = 1.5



(d) c = 2.5

Figure 4: Nonlinear wave profiles of solution (39) for different values of c.

4.2 Limitation

- This method can have restrictions for nonlinear partial differential equation in the fields such as complex fluid dynamics, turbulence, and strongly nonlinear PDEs.
- Sometime the results of algebraic equation are long and complicated,which can long time to evaluate the solutions.

5 Conclusion

In this work, we investigated several well-known nonlinear PDEs through exponential rational function method. We obtained the exact solutions for the all investigated equations. We utilized a rational function in term of the exponential functions, which is suitable for the study of soliton solutions, that gave different nonlinear wave solutions. Due to exponential rational forms, we obtained the exact solutions of the explored nonlinear PDEs. We investigated the ERFM to the well-known equations such as KdV equation, KP equation, mKdV equation, and KMN equation. Also, we analysed the dynamical behavior for the obtained solutions in three dimensional graphics for appropriate values of the arbitrary parameters. The studied models explains the nonlinear wave phenomena from different fields such as fluid dynamics, oceanography, plasma physics, opitic fibers, and other sciences.

Declarations

Ethics approval and consent to participate

Not applicable.

Conflict of interest

The author claims that there are no conflicts of interest.

Data availability statement

No datasets have been generated or analyzed during the current investigation.

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